Lesson Plan: Graphing Cubic Functions

Subject: Mathematics
Course: IB Mathematics Analysis and Approaches
Level: IB HL
Topic: Graphing Cubic Functions
Duration: 80 minutes

Learning Objective:

By the end of this lesson, students will be able to:

- Understand the key features of cubic functions and their graphs.
- Explore the transformations of the basic cubic function $f(x) = x^3$.
- Analyze and graph different forms of cubic functions, including factored forms.
- Use given points and intercepts to construct the equation of a cubic function.

Resources & Materials

- **PowerPoint slides**: "AAH2.12.8 Graphing cubic functions.pptx."
- **Whiteboard / interactive board** for note-taking and illustrating expansions.
- **Student handouts** (optional) with guiding questions and practice problems.
- **Graphing calculators** or other CAS (optional for verification).

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1. Lesson Introduction (10 minutes)

Engagement – Inquiry Prompt:

Pose the following question to spark student thinking:

"What does the graph of a cubic function look like, and how can we tell what it will do just by looking at the equation?"

Prompts for small group discussion:

- What happens to $f(x) = x^3$ as $x \to \infty$ or $x \to -\infty$?
- What is the effect of multiplying the function by a positive or negative constant?
- What might change if we shift the function left, right, up, or down?

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2. Exploring the Basic Cubic Graph (15 minutes)

- Students sketch $y = x^3$ by hand or using technology for values from x = –3 to 3.
- Discuss symmetry, shape, and end behaviour.
- Introduce transformations:
  - Vertical stretch/compression: $y = a \cdot x^3$
  - Horizontal/vertical shifts: $y = (x \pm b)^3 + c$
  - Reflections over axes

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3. Interactive Graphing with Technology – (20 minutes)

Students use a graphing calculator or software to investigate:

$y = x^3$

- $y = 2x^3$, $y = -x^3$, $y = (x - 2)^3$, $y = x^3 + 1$, etc.

Students record and describe the transformations in pairs.

Guiding Question: How does each coefficient or constant affect the shape and position of the graph?

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4. Understanding Roots and Graph Shapes (15 minutes)

Explore different types of cubic graphs based on their factored form:

- Three distinct real roots
- One real double root and one single root
- Triple real root
- One real root, complex conjugate pair

Sketch examples for each and connect to algebraic structure.

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5. Constructing a Cubic Function from a Graph (15 minutes)

Given intercepts and one point, students build a function:

Example: x-intercepts at –1, 2, and 4; point (0, -8)

Step-by-step guidance on forming $f(x) = a(x + 1)(x - 2)(x - 4)$ and solving for a.

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6. Exit Ticket & Reflection (5 minutes)

Exit Ticket Prompts:

- What determines the shape of a cubic function's graph?
- How do we recognize transformations from the equation?
- What does the sign of the leading coefficient tell us?

Assessment & Differentiation

Assessment:

- Observation of graphing tasks
- Accuracy of descriptions and equations formed
- Exit ticket reflections

Differentiation:

- Support students with graph templates or pre-labeled points
- Challenge advanced learners with more abstract forms (e.g., complex roots)