# Worksheet: The Factor Theorem

**Student Name:** ______________________________________ **Date:** _______________

**Learning Objective:**

Apply the Factor Theorem to identify and factorize polynomials.

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## Part A: Conceptual Understanding

1. Define the Factor Theorem in your own words. Explain how it relates to the Remainder Theorem.
2. Suppose you have a polynomial $f(x)$. Describe two methods to check if $(x - 1)$ is a factor of $f(x)$, and discuss any advantages or disadvantages.

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## Part B: Direct Application

3. Show that $(x + 2)$ is a factor of $f(x) = 3x^2 + 5x - 2$. Hence or otherwise, fully factorize the polynomial.
4. If a polynomial $P(x) = x^3 - 3x^2 - 6x + 8$, show that $x + 1$, is a factor. Factorize $P(x)$ completely.
5. Find all factors of the polynomial $g(x) = x^3 + x^2 - 4x - 4$ using the Factor Theorem.

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## Part C: Solving for Unknown Coefficients

6. Let $P(x) = x^3 + 4x^2 + ax + b$. If $x - 2$is a factor and $P(3) = 0$, determine the values of $a$ and $b$.
7. Consider the polynomial $h(x) = 2x^3 + px^2 + qx + r$. It is known that $(x - 1)$ is a factor $h(-3) = 0$ and $h(2) = 10$. Find all possible values for $p, q, r$ .

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## Part D: Extension Challenge

8. For $P(x) = x^3 - 8$, use the Factor Theorem to find a linear factor. Then, factorize $P(x)$ fully.
9. If a polynomial $P(x)$ of degree 4 has known factors $(x - 1)$ and $(x + 2)$, outline a general process to find its remaining factors using the Factor Theorem

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## Instructions for Submission:

- Solve the questions on a separate sheet or digitally and submit them by the given deadline.
- Justify all answers with proper reasoning.

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**End of Worksheet**

**Solutions**

## Part B: Direct Application

3. Evaluate $f(-2) = 3(-2)^2 + 5(-2) - 2 = 12 - 10 - 2 = 0$

   Therefore, $x + 2$ is a factor.

   Divide $3x^2 + 5x - 2$ by $x + 2$ (using synthetic or long division):

   $3x^2 + 5x - 2 = (x + 2)(3x - 1)$.

4. Evaluate $P(1) = 1 - 3 - 6 + 8 = 0$ Therefore, $x - 1$ is a factor.

   Factorizing $x^3 - 3x^2 - 6x + 8$ by dividing by $x - 1$ yields $(x - 1)(x^2 - 2x - 8)$.

   Factor $x^2 - 2x - 8 = (x - 4)(x + 2)$.

   Thus, $P(x) = (x - 1)(x - 4)(x + 2)$.

5. Evaluate possible integer roots (e.g., $x = 1, -1, 2, -2, 4, -4$ etc.) until a zero is found.

   Example: $g(1) = 1 + 1 - 4 - 4 = -6$ (not zero), $g(2) = 8 + 4 - 8 - 4 = 0$. So $x - 2$ is a factor.

   Continue with polynomial division to find remaining factors.

## Part C: Solving for Unknowns

6. **Solution**:
   - Since $x - 2$ is a factor, $f(2) = 0$. So $2^3 + 4(2)^2 + 2a + b = 0$. That is $8 + 16 + 2a + b = 0$.
   - Also, $f(-3) = 0$. Thus $(-3)^3 + 4(-3)^2 - 3a + b = 0 \Rightarrow -27 + 36 - 3a + b = 0$.
   - From these two equations, solve for $a$ and $b$.
7. **Solution**:
   - Since $x - 1$ is a factor, $h(1) = 0 \therefore 2(1)^3 + p(1)^2 + q(1) + r = 0$
     $\therefore 2 + p + q + r = 0$. (Equation (1))
   - Also, $h(2) = 10 \therefore 2(2)^3 + p(2)^2 + q(2) + r = 0$
     $\therefore 16 + 4p + 2q + r = 10$. (Equation (2))
   - Solve these simultaneous equations:
   - Express $r$ in terms of $p$ and $q$.
     From equation 1: $r = -2 - p - q$.
     **Substitute this into the second equation**
     Substitute $r = -2 - p - q$ into Equation (2):
     $16 + 4p + 2q + (-2 - p - q) = 10$.
     Simplify:
     $16 - 2 + 4p - p + 2q - q = 10 \Longrightarrow 14 + 3p + q = 10$.
     Hence,

$3p + q = -4$. Equation (3)

**Solve for *q* and *r* in terms of *p***

From Equation (3):

$q = -4 - 3p$.

Then from Equation (2) $r = -2 - p - q$:

$r = -2 - p - (-4 - 3p) = -2 - p + 4 + 3p$

$r = 2 + 2p$.

**Combine the results**

We see that $p$ can be any real number. For each chosen $p$, we get:

$q = -4 - 3p$, $r = 2 + 2p$.

Therefore, **the general solution** for $(p, q, r)$ is:

$(p, -4 - 3p, 2 + 2p)$,

where $p$ is any real number.

**Part D: Challenge**

**8. Identify a potential root using the Factor Theorem**

The expression $x^3 - 8$ can be rewritten as $x^3 - 2^3$.

- Intuitively, we might suspect $x = 2$ could be a root because $2^3 = 8$.

**Check if x = 2 is indeed a root**

Evaluate $P(2)$:

$P(2) = (2)3 - 8 = 8 - 8 = 0$.

Since $P(2) = 0$, it follows from the Factor Theorem that $(x-2)$ is a factor of $P(x)$

**Perform factorization using the difference of cubes**

Recall the **difference of cubes** formula:

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

In this case, $a = x$ and $b = 2$. So we can write:

$x^3 - 2^3 = (x - 2)(x^2 + 2x + 4)$.

Check if the remaining quadratic can be factored further

- Since the discriminant is negative, there are no real roots, and $x^2 + 2x + 4$ remains irreducible over the real numbers.

**Conclusion**

The **fully factorized form** of $P(x)$ over the real numbers is:

$P(x) = (x - 2)(x^2 + 2x + 4)$.

9. Let the polynomial be $P(x)$. Since it's degree 4 and you know two linear factors, you can write: $P(x) = (x - 1)(x + 2)\,Q(x)$, where $Q(x)$ is a quadratic polynomial of the form $Q(x) = ax^2 + bx + c$.
   Set Up the Division (or Synthetic Division)
   If the coefficients of $P(x)$ (the original polynomial) are known, you can divide P(x) by each factor separately or divide by the product $(x - 1)(x + 2)$ directly.
   After this division, you will be left with a quadratic quotient $Q(x)$.
   Apply the Factor Theorem to the Quotient
   Once you have $Q(x) = ax^2 + bx + c$, you can attempt to further factorize it:
   Check the discriminant $\Delta = b^2 - 4ac$.
   If $\Delta > 0$, you can factor over the real numbers (two distinct real roots).
   If $\Delta = 0$, the quadratic has a repeated real root (perfect square factorization).
   If $\Delta < 0$, the quadratic is irreducible over the real numbers (it can only be factored over the complex numbers).
   Conclude the Complete Factorization
   Combine $(x - 1)$, $(x + 2)$, and your factorization (or irreducible form) of $Q(x)$ to express the complete factorization of $P(x)$.
   Symbolically: $P(x) = (x - 1)(x + 2)$ (factorized form of $Q(x)$).