

Subject: Mathematics

Course: IB Mathematics Analysis and Approaches

Level: IB HL

Topic: The Factor Theorem

Duration: 80 minutes

Learning Objective:

Students will understand and apply the **Factor Theorem** to determine factors of polynomials and perform polynomial factorization.

1. Lesson Introduction (10 minutes)

Engagement – Inquiry Prompt:

- Display the polynomial: $f(x) = x^3 - 3x^2 - 6x + 8$
- Ask: “Can we determine if $x - 1$ is a factor of this polynomial without performing full polynomial division?”
- Encourage students to suggest a method to test for divisibility.
- Introduce the **Factor Theorem**: “If $f(p) = 0$, then $x - p$ is a factor of $f(x)$.”

Objective Setting:

- Highlight that the Factor Theorem is an extension of the Remainder Theorem and is a useful tool for polynomial factorization.

2. Exploration – Guided Discovery (20 minutes)

Activity: Small Group Investigation

- Divide students into small groups and provide them with three different polynomial to explore factorization.
- Task:
 - Evaluate $f(x)$ at different integer values to find possible factors.
 - Once a factor is found, use polynomial division or synthetic division to fully factorize the polynomial.
 - Verify the factorization by expanding the factors back.
- **Inquiry Questions:**
 - What do you notice when $f(p) = 0$?
 - How can you check if your factorization is correct?
 - What patterns emerge when testing for factors?

Class Discussion:

- Each group presents their findings and explains their reasoning.
- Address misconceptions and clarify how the theorem simplifies polynomial division.

3. Application – Extending Understanding (25 minutes)**Problem-Solving Scenarios:**

- Work through Example from the provided PowerPoint:
 - Show that $x + 2$ is a factor of $f(x) = 3x^2 + 5x - 2$ and use it to fully factorize the polynomial.
- **Inquiry Approach:**
 - Students attempt independently for 5 minutes.
 - Discuss methods in pairs.
 - Teacher scaffolds reasoning without providing direct solutions.
 - Whole-class walkthrough.

Challenge Question:

- Given , $P(x) = x^3 + 4x^2 + ax + b$ if $x - 2$ and $x + 3$ are factors, find a and b .

4. Reflection and Consolidation (15 minutes)**Student-Generated Questions:**

- Ask students to write one question they still have about the theorem.
- Discuss selected questions to clear misunderstandings.

Exit Ticket:

- Solve: Show that $x - 1$ is a factor of $x^3 - 3x^2 - 6x + 8$ and find the complete factorization.
- One-minute written reflection: *"How does the Factor Theorem simplify polynomial division?"*

Assessment & Homework

- **Formative Assessment:**

- Class participation in group work and discussion.
- Accuracy in factorizing polynomials using the theorem.

- **Homework:**

- Apply the Factor Theorem to different polynomials.
- Explain the theorem in your own words and create an example.

Differentiation

- For advanced students: Introduce the relationship between the Factor Theorem and the Fundamental Theorem of Algebra.
- For struggling students: Provide additional worked examples and use visual step-by-step guide for synthetic division.