Worksheet: The Remainder Theorem

Student Name: ________________________________ **Date:** ____________

Learning Objective:

Apply the Remainder Theorem to evaluate polynomials, determine remainders, and solve equations involving unknown coefficients.

Part A: Conceptual Understanding

1. Explain in your own words what the Remainder Theorem states. Use an example to support your explanation.
2. When a polynomial $f(x)$ is divided by $x - p$, why does evaluating $f(p)$ give the remainder?

Part B: Direct Application

Solve the following problems using the Remainder Theorem. Show all steps.

3. Find the remainder when $f(x) = 2x^3 - 5x^2 + 7x - 3$ is divided by $x - 2$.
4. If a polynomial $P(x) = x^4 - 3x^3 + x - 4$ is divided by $x + 1$, find the remainder.
5. Find the remainder when $g(x) = 4x^4 + 2x^3 - 6x + 9$ is divided by $x - 3$.

Part C: Solving for Unknown Coefficients

6. When the polynomial $P(x) = x^3 + 4x^2 + ax + b$ is divided by $x - 2$, the remainder is 10. When divided by $x + 3$, the remainder is -4. Find the values of a and b.
7. The polynomial $Q(x) = x^4 + kx^2 + 2x + 3$ leaves a remainder of 5 when divided by

   $x - 1$. Find the value of k.

Part D: Extension Challenge

8. Suppose $R(x) = x^3 + px^2 + qx + r$ leaves a remainder of 7 when divided by $x - 1$ and a remainder of -3 when divided by $x + 2$. Find the possible values of p, q, r.
9. If a polynomial $S(x)$ is divided by both $x - 1$ and $x + 2$, the sum of the remainders is always equal to 5.

   Determine whether this is always true for any polynomial of degree 3.

Instructions for Submission:

- Solve the questions on a separate sheet or digitally and submit them by the given deadline.
- Justify all answers with proper reasoning.

End of Worksheet

Solutions

Part B: Direct Application

3. $f(2) = 2(2)^3 - 5(2)^2 + 7(2) - 3 = 16 - 20 + 14 - 3 = 7$
4. $P(-1) = (-1)^4 - 3(-1)^3 + (-1) - 4 = 1 + 3 - 1 - 4 = -1$
5. $g(3) = 4(3)4 + 2(3)3 - 6(3) + 9 = 324 + 54 - 18 + 9 = 369$

Part C: Solving for Unknowns

6. Given $P(2) = 10$ and $P(-3) = -4$, setting up the equations:
   - $2^3 + 4(2)^2 + 2a + b = 10$
   - $(-3)^3 + 4(-3)^2 - 3a + b = -4$
     Solving simultaneously gives $a = -5$, $b = 6$.
7. $Q(1) = (1)^4 + k(1)^2 + 2(1) + 3 = 5$, solving for k gives $k = -1$.

Part D: Challenge

8. Setting up equations using given remainders and solving simultaneously gives the possible values for p, q, r.
9. Exploring the divisibility condition for polynomials of degree 3 and confirming the sum condition.