Division of Polynomials

Student Name: _____ **Date:** _____

Section A: Long Division of Polynomials

1. Perform the following polynomial divisions using long division. Express your answer as:
   Dividend = (Divisor) × (Quotient) + Remainder

a) $(2x^3 - 3x^2 + 4x + 1) \div (x - 2)$

Answer: _____

b) $(x^3 - x^2 - 7x + 3) \div (x - 3)$

Answer: _____

c) $(x^4 - 4x^3 + 2x + 3) \div (x^2 - x + 1)$

Answer: _____

Section B: Factor Test & Interpretation

2. Determine whether the divisor is a factor of the polynomial.

a) Is $x - 3$ a factor of $x^3 - x^2 - 7x + 3$?
Answer: _____

b) Is $x + 1$ a factor of $x^3 + 2x^2 + x + 1$?
Answer: _____

Section C: Division by Equating Coefficients

3. Let $f(x) = 3x^2 + 11x - 8$ and divide it by $x + 5$ using equating coefficients.
   Write an identity:
   $3x^2 + 11x - 8 \equiv (x + 5)(ax + b) + r$
   Solve for a, b, and r.

Answer:
   a = _____, b = _____, r = _____

Section D: Conceptual Understanding

4. Consider the division:
   $f(x)/g(x) = q(x) + r(x)/g(x)$

a) What are the conditions for the remainder to be 0?
Answer: _____

b) Why must the degree of $r(x)$ be less than the degree of $g(x)$?
Answer: _____

Section E: Challenge Problem

5. Create a cubic polynomial f(x) such that when divided by x - 2, the remainder is 5 and the quotient is $2x^2 + x + 2$.

   Use the division algorithm:

   f(x) = (x - 2)(2x² + x + 2) + 5

Answer: _____

Bonus: Real-World Application

6. A rocket's altitude is modeled by the polynomial:

   h(x) = 3x³ + 5x² - 2x + 7

a) Divide h(x) by x + 1 using long division.
Answer: _____

b) Interpret the quotient and remainder in the context of time and height.
Answer: _____

Solutions

Section A Solutions

1a. $(2x^3 - 3x^2 + 4x + 1) \div (x - 2) = (2x^2 + x + 6)(x - 2) + 13$

1b. $(x^3 - x^2 - 7x + 3) \div (x - 3) = (x^2 + 2x - 1)(x - 3)$

1c. $(x^4 - 4x^3 + 2x + 3) \div (x^2 - x + 1) = (x^2 - 3x - 4)(x^2 - x + 1) + (x - 7)$

Section B Solutions

2a. Remainder is 0 → x - 3 is a factor.

2b. Remainder is not 0 → x + 1 is not a factor.

Section C Solutions

a = 3, b = -4, r = 12

Final: $3x^2 + 11x - 8 = (x + 5)(3x - 4) + 12$

Section D Solutions

4a. The remainder is 0 when the divisor is a factor of the dividend.

4b. The degree of r(x) must be less than g(x) to ensure uniqueness of q(x) and r(x).

Section E Solutions

$f(x) = (x - 2)(2x^2 + x + 2) + 5 = 2x^3 - 3x^2 + x + 1$

Bonus Solutions

6a. $h(x) \div (x + 1) = (3x^3 + 5x^2 - 2x + 7)/(x + 1) = 3x^2 + 2x - 4 + 11/(x + 1)$

6b. Quotient represents the rocket's changing altitude rate; remainder reflects offset in model.