Polynomials and Operations with Polynomials

Student Name: ___________________________________ Date: ___________

Section 1: Concept Exploration

1. **Polynomial Classification**
   Identify whether each of the following expressions is a polynomial. If it is, state its degree and leading coefficient.
   (a) $3x^4 - 5x^2 + 2$
   (b) $\frac{2}{x} + x^3 + 7$
   (c) $5x^2 + \sqrt{x} - 3$
   (d) $-2x^5 + 4x^3 - x + 6$

Section 2: Operations with Polynomials

2. **Adding and subtracting Polynomials**

   Perform the following operations and simplify your answers:

   (a) $(2x^3 - 4x^2 + 5x - 3) + (x^3 + 2x^2 - 7x + 6)$

   (b) $(4x^4 - 3x^2 + 7) - (2x^4 + 5x^2 - 4x + 2)$

3. **Multiply Polynomials**

   Expand and simplify:

   (a) $(x + 2)(x - 3)$

   (b) $(2x^2 + x - 4)(x - 5)$

   (c) $(x^3 - 2x + 4)(2x^2 + 3x - 5)$

Section 3: Real-World Applications

4. **Application in Motion**

   A ball is thrown into the air with a height modeled by the polynomial function: $h(x) = -2x^2 + 5x + 10$

   (a) What type of polynomial is this? State its degree and leading coefficient.
   (b) If another object follows a path given by $h_2(x) = -x^2 + 3x + 8$ find the sum $h(x) + h_2(x)$ and interpret its meaning.

5. **Investment Growth Model**

   A company models its revenue growth with the polynomial function $R(x) = 3x^3 - 5x^2 + 2x + 7$

   Another company has a revenue model: $S(x) = -2x^3 + 4x^2 - x + 5$

   (a) Find the combined revenue function $R(x) + S(x)$.
   (b) What does the degree of the resulting polynomial tell us about long-term growth?

Section 4: Challenge Problems

6. **Exploring Degree & Growth**

   (a) If $P(x) = ax^n + ax^{n-1} + \ldots + k$ represents a polynomial, explain why multiplying two polynomials always results in a new polynomial.

   (b) Predict the degree of $P(x) \times Q(x)$.if $P(x)$ has degree 4 and $Q(x)$ has degree 3.

7. **Polynomial Puzzle**

Given that $P(x)$ is a polynomial such that $P(x) \times (x - 1) = x^4 - x^3 + 2x - 3$. Determine $P(x)$ and verify your answer.

Reflection Question

8. **Critical Thinking**

In your own words, explain how the structure of a polynomial changes when performing different operations. How does this impact real-world problem-solving.

Bonus Inquiry Task

9. **Create your own Problem**

Think of a real-world scenario where polynomial operations would be useful (E.g. physics, economics, engineering). Write a problem involving polynomials and solve it.

Solutions

Section 1: Concept Exploration

1. Polynomial Classification

a) $3x^4 - 5x^2 + 2$

✅ Polynomial

- Degree: 4
- Leading Coefficient: 3

d) $-2x^5 + 4x^3 - x + 6$

✅ Polynomial

- Degree: 5
- Leading Coefficient: –2

Section 2: Operations with Polynomials

2. Adding and Subtracting Polynomials

a) Addition

$(2x^3 - 4x^2 + 5x - 3) + (x^3 + 2x^2 - 7x + 6)$
$= 3x^3 - 2x^2 - 2x + 3$

b) Subtraction

$(4x^4 - 3x^2 + 7) - (2x^4 + 5x^2 - 4x + 2)$
$= 2x^4 - 8x^2 + 4x + 5$

3. Multiplying Polynomials

a) $(x + 2)(x - 3) = x^2 - x - 6$

b) $(2x^2 + x - 4)(x - 5) = 2x^3 - 9x^2 - 9x + 20$

c) $(x^3 - 2x + 4)(2x^2 + 3x - 5) = 2x^5 + 3x^4 - 9x^3 + 2x^2 + 22x - 20$

Section 3: Real-World Applications

4. Application in Motion

a) Height function:

$h(x) = -2x^2 + 5x + 10$

- Type: Quadratic Polynomial
- Degree: 2
- Leading Coefficient: –2

b) Combined height:

$h(x) + h_2(x) = (-2x^2 + 5x + 10) + (-x^2 + 3x + 8) = -3x^2 + 8x + 18$ is the combined height

5. Investment Growth Model

(a) Combined revenue function:

$R(x) + S(x) = (3x^3 – 5x^2 + 2x + 7) + (–2x^3 + 4x^2 – x + 5) = x^3 − x^2 + x + 12$

b) Interpretation of degree:

The resulting polynomial has degree 3, indicating that in the long term, the cubic term dominates, leading to accelerated growth compared to lower-degree terms.

Section 4: Challenge Problems

6. Exploring Degree & Growth

a) Why does the product of polynomials yield another polynomial?
Because multiplying polynomials involves distributing and combining terms, all resulting terms remain in the form ax^n, which meets the definition of a polynomial.

b) Degree of product ($P(x) \times Q(x)$):
If deg(P) = 4 and deg(Q) = 3, then deg(PQ) = 7

7. Polynomial Puzzle:
Given $P(x)(x – 1) = x^4 – x + 2x – 3$,
Then
$P(x) = \frac{x^4-x^3+2x-3}{x-1}$
This simplifies to the quotient of the polynomial division (can be performed via long division if needed).

d) Reflection:
The structure of a polynomial changes based on the operation:

- Addition/Subtraction: changes coefficients, preserves degree unless leading terms cancel.
- Multiplication: increases the degree and number of terms. This matters in real-world modeling because the degree influences growth rate and behavior over time.