# Worksheet: Disproof by Counterexample and Proof by Contrapositive

**Name: ________________________________________ Date: __________________**

## Section 1: Disproof by Counterexample

Find a counterexample that disprove the following statements:

1: "For all integers $n$, $n^2 + n$ is divisible by 3."

2: "The product of two prime numbers is always odd."

3: "For all real numbers $x$, $|x| \geq x^2$."

4: "The square of any integer is always greater than or equal to the integer."

5: "The product of two even numbers is always divisible by 8."

## Section 2: Proof by Contrapositive

Prove by constructing the contrapositive the following statements

1: If $n^2$ is even, then $n$ is even.

2: If a product $ab = 0$, then either $a = 0$ or $b = 0$.

3: If a number is not divisible by 3, then its square is not divisible by 3.

4: If $n^2$ is divisible by 4, then $n$ is even.

5: If a number is odd, then its square is also odd.

## Section 1: Disproof by Counterexample

1: The statement "For all integers $n$, $n^2 + n$ is divisible by 3" is false.

Counterexample: Let $n = 1$. $n^2 + n = 1^2 + 1 = 2$, which is not divisible by 3.

2: The statement "The product of two prime numbers is always odd" is false.

Counterexample: 2 and 3 are prime numbers, but their product is 6, which is even.

3: The statement "For all real numbers $x$, $|x| \geq x^2$" is false.

Counterexample: Let $x = 0.5$. Then $|x| = 0.5$, but $x^2 = 0.25$. Here, $|x| < x^2$.

4: The statement "The square of any integer is always greater than or equal to the integer" is false.

Counterexample: Let n = 0. Then n^2 = 0, which equals n but does not satisfy the inequality.

5: The statement "The product of two even numbers is always divisible by 8" is false.

Counterexample: Let the numbers be 2 and 4. Their product is 8, which is divisible by 8. However, if we take 2 and 6, their product is 12, which is not divisible by 8.

## Section 2: Proof by Contrapositive

1: Prove: If $n^2$ is even, then $n$ is even.

Contrapositive: If $n$ is odd, then $n^2$ is odd.

Proof: If $n$ is odd, let $n = 2k + 1$ for some integer $k$.

Then $n^2 = (2k + 1)^2 = 4k^2 + 4k + 1$, which is odd.

Thus, the contrapositive is true, and the original statement is proven.

2: Prove: If $ab = 0$, then either $a = 0$ or $b = 0$.

Contrapositive: If $a \neq 0$ and $b \neq 0$, then $ab \neq 0$.

Proof: If $a \neq 0$ and $b \neq 0$, their product $ab$ is not zero.

Hence, the contrapositive is true, and the original statement is proven.

3: Prove: If a number is not divisible by 3, then its square is not divisible by 3.

Contrapositive: If a number's square is divisible by 3, then the number is divisible by 3.

Proof: Let $n^2$ be divisible by 3. Then $3 \mid n^2$. Since 3 is a prime, it follows that $3 \mid n$. Thus, the contrapositive is true, and the original statement is proven.

4: Prove: If $n^2$ is divisible by 4, then $n$ is even.

Contrapositive: If $n$ is odd, then $n^2$ is not divisible by 4.

Proof: If $n$ is odd, let $n = 2k + 1$ for some integer $k$. Then $n^2 = (2k + 1)^2 = 4k^2 + 4k + 1$, which is not divisible by 4.

Thus, the contrapositive is true, and the original statement is proven.

5: Prove: If a number is odd, then its square is also odd.

Contrapositive: If a number's square is even, then the number is even.

Proof: If $n^2$ is even, then let $n = 2k$ for some integer $k$.

The square of an even number is also even.

Thus, the contrapositive is true, and the original statement is proven.