

Subject: Mathematics

Course: IB Mathematics Analysis and Approaches

Level: IB HL

Topic: Disproof by Counterexample and Proof by Contrapositive

Duration: 60 minutes

Lesson Objectives

By the end of the lesson, students will be able to:

1. Understand and use the method of disproof by counterexample.
2. Understand and apply proof by contrapositive in mathematical reasoning.
3. Develop critical thinking and collaborative problem-solving skills.

Resources

- **PowerPoint Presentation:** ["Disproof by Counterexample and Proof by Contrapositive"]
- Whiteboard and markers
- Worksheets with examples for group activity
- Calculator (if required).

Lesson Outline

1. Introduction (10 minutes)

Objective: Engage students and introduce the concepts.

- **Starter Question:**

Pose the question: *"How can we disprove a mathematical statement effectively?"*

- Allow students to brainstorm for 2-3 minutes.

- **Explain Disproof by Counterexample:**

- Use Slide 2 of the PowerPoint: Explain that a single counterexample can disprove a universal statement.
- Highlight the steps:
 1. Find a single example that contradicts the statement.
 2. Show why this example disproves the statement.
 3. Conclude the general statement is false.

- **Transition to Proof by Contrapositive:**

- Use Slides 7 - 10 to define contrapositive and its logical equivalence to the original statement.
- Highlight its structure:
 1. State the contrapositive: If not Q, then not P.
 2. Assume Q is false.
 3. Prove that P is false.

2. Guided Exploration (20 minutes)

Objective: Work through examples together as a class.

Part 1: Disproof by Counterexample

- Example 1: (Slide 3) Disprove the statement: "All prime numbers are odd."
 - Walk through the steps:
 1. Identify $n = 2$ as a prime number.
 2. Show that $n = 2$ is even.
 3. Conclude the statement is false.
- Example 2: (Slide 4) Disprove the statement: "For all real numbers x , $x^2 + 2x + 1 \geq 1$."
 - Walk through the steps:
 1. Substitute $x = 0$ into the expression.
 2. Show the result $(-1)^2 + 2(-1) + 1 = 0$, which is less than 1
 3. Conclude the statement is false.
- Example 3 (Slide 5) Disprove the statement: "For all integers n , $n^3 + n$ is divisible by 4."
 - Substitute $n = 2$ into the expression.
 - Show the result $2^3 + 2 = 10$, which is not divisible by 4.
 - Conclude the statement is false.

Part 2: Proof by Contrapositive

- Example 1: (Slide 11) Prove: "If n^2 is even, then n is even."
 - Contrapositive: "If n is odd, then n^2 is odd."
 - Assume $n = 2k + 1$.
 - Compute $n^2 = (2k + 1)^2 = 4k^2 + 4k + 1$, which is odd.
 - Conclude that the contrapositive is true, so the original statement is true.
- Example 2: (Slide 12) Prove: "If $n^2 - 4n + 5$ is even, then n is odd."
 - Contrapositive: "If n is even, then $n^2 - 4n + 5$ is odd."
 - Assume $n = 2k$ and compute $n^2 - 4n + 5 = 4k^2 - 8k + 5$, which is odd.
 - Conclude that the contrapositive is true, so the original statement is true.

3. Inquiry-Based Activity (20 minutes)

Objective: Apply both methods to new problems in small groups.

- Activity Setup:
- Divide students into groups of 3-4 and assign one of the following problems:
 1. Disprove the statement: "All square numbers are divisible by 4."
 2. Prove: "If n is not divisible by 3, then n^2 is not divisible by 9" using contrapositive.

Instructions:

1. Identify the method required: disproof by counterexample or proof by contrapositive.
2. Work collaboratively to solve the problem.
3. Present findings to the class in 2-3 minutes.

Teacher's Role:

- Facilitate group work and provide guidance.
- Ask guiding questions:
 - "How can you find a counterexample?"
 - "What would the contrapositive of this statement look like?"

4. Conclusion and Reflection (10 minutes)

Objective: Reflect on learning and address misconceptions.

- Class Discussion:
 - What was the difference between disproof by counterexample and proof by contrapositive?
 - How does a counterexample show a statement is false?
 - Why is the contrapositive logically equivalent to the original statement?
- Reflection Questions:
 - What challenges did you face during the group activity?
 - How can you use these methods in other areas of mathematics?
- Key Takeaways:
 - Disproof by counterexample requires only one example to disprove a universal statement.
 - Proof by contrapositive uses logical equivalence to simplify proofs.

Assessment and Homework

- Evaluate group presentations for accuracy and clarity.
- Work on the worksheet problems