

Subject: Mathematics

Course: IB Mathematics Analysis and Approaches

Level: IB HL

Topic: Proof by Exhaustion

Duration: 60 minutes

Lesson Objectives

By the end of the lesson, students will be able to:

1. Understand the concept of proof by exhaustion and its application.
2. Use proof by exhaustion to solve mathematical problems.
3. Develop critical thinking and inquiry skills by exploring multiple cases.

Resources

- **PowerPoint Presentation:** [Proof by Exhaustion Presentation]
- Whiteboard and markers
- Student worksheets
- Calculator (if required).

Lesson Outline

1. Introduction (10 minutes)

Objective: Engage students with the concept of proof by exhaustion.

- **Starter Activity:**
 - Pose the question: *"How do we prove something is true in all cases when there are multiple possibilities?"*
 - Allow students to brainstorm for 2-3 minutes and share their ideas.
- **Teacher's Explanation:**
 - Introduce the concept of **Proof by Exhaustion** using Slide 2 of the PowerPoint.
 - Highlight the steps: dividing the problem into finite cases, proving each case, and concluding.

2. Guided Practice (20 minutes)

Objective: Explore examples of proof by exhaustion with teacher guidance.

- Example 1 (Slide 4): *Prove that the square of an even number is divisible by 4.*
 - Walk students through the example step by step:
 - Explain the setup: a number $n = 2k$.
 - Discuss the only cases:
 - Conclude the divisibility.

IB A&P • Example 2 (Slide 5): *Prove that $n(n + 1)$ is even for all integers n .*

- Ask students to explain why $n = 2k$ represents an even number.
- Ask students to explain how to represent an odd number.
- Example 3 (Slide 6): *Given that $n^5 - n = (n - 1)n(n + 1)(n^2 + 1)$*

Prove that $n^5 - n$ is divisible by 5 for all $n \in \mathbb{Z}$.

- Ask students to think about all the possible cases.
- Class Discussion:
 - What makes these proofs exhaustive?
 - How do we ensure we have covered all possible cases?

3. Inquiry-Based Activity (20 minutes)

Objective: Students apply proof by exhaustion to new problems.

- Activity Setup:
 - Divide students into small groups and assign them one or two of the problems from the worksheet:.
- Instructions:
 - Groups must:
 1. Identify the possible cases.
 2. Prove each case thoroughly.
 3. Summarize their findings on a mini whiteboard or paper.
- Teacher's Role:
 - Circulate and provide guidance to groups.
 - Ask probing questions like:
 - "Have you considered all cases?"
 - "How can you be sure your proof is exhaustive?"

4. Conclusion and Reflection (10 minutes)

Objective: Consolidate understanding and reflect on the inquiry process.

- Class Discussion:
 - Invite each group to present their solution briefly.
 - Discuss similarities and differences in their approaches.

- Highlight the importance of covering all cases in a proof by exhaustion.
- Reflection Questions:
 - What challenges did you face when identifying all possible cases?
 - How does proof by exhaustion differ from other proof methods (e.g., direct proof, induction)?
 - In what real-life scenarios might you use exhaustive proof techniques?

Assessment and Homework

- Classwork: Monitor group work for understanding and completeness.
- Finish any incomplete worksheet problems.