Lesson Plan: Proof by Contradiction

Subject: Mathematics
Course: IB Mathematics Analysis and Approaches
Level: IB HL
Topic: Proof by Contradiction
Duration: 60 minutes

Lesson Objectives

By the end of the lesson, students will be able to:

- Understand the concept and steps of proof by contradiction (Reductio ad Absurdum).
- Apply proof by contradiction to solve mathematical problems.
- Develop logical reasoning and critical thinking skills.

Resources

- PowerPoint Presentation**:** [Proof by Contradiction Presentation]
- Whiteboard and markers
- Student worksheets
- Calculator (if required).

Lesson Outline

1. Introduction (10 minutes)

Objective: Build students' curiosity and activate prior knowledge.

- **Starter Activity:**
  Write on the board:
  - "Can we prove something by assuming the opposite is true?"
  - Allow students to brainstorm for 2-3 minutes and share their ideas.
- **Teacher's Explanation:**

Use the PowerPoint slide to define proof by contradiction and outline its steps:

1. Assume the negation of the statement you want to prove.
2. Show that this assumption leads to a logical contradiction.
3. Conclude that the original statement must be true.

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2. Group Exploration: Example 1 (15 minutes)

Objective: Guide students through a structured example.

Activity:

- Work through the proof that $\log_2(3)$ is irrational using the steps provided in the presentation.
- Allow students to suggest logical steps and write them on the board.
- Highlight the logical contradiction when p and q have common factors.

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3. Guided Practice: Example 2 (10 minutes)

Objective: Encourage collaboration and shared reasoning.

- Activity:
  - Use the proof that there are infinitely many prime numbers as an example.
  - Let students work in pairs to follow the steps:
    1. Assume there are finitely many primes.
    2. Construct $N = p_1 \times p_2 \times \cdots \times p_n + 1$.
    3. Discuss why N leads to a contradiction.

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4. Independent Inquiry: Example 3 (10 minutes)

- Objective: Build independent problem-solving skills.
- **Activity**:
  - Provide students with the following question:
    Prove that the square root of 2 is irrational using proof by contradiction.
  - Students work individually to complete the steps.
  - Circulate to provide guidance as needed.

5. Reflection and Discussion (5 minutes)

- **Objective:** Reinforce learning through reflection.
- **Activity:**
  - Ask students: "What did you find challenging about proof by contradiction?"
  - Discuss its importance in mathematics and why it is a powerful proof method.

6. Homework Assignment

Distribute the worksheet with a variety of proof by contradiction problems for additional practice**.**