Lesson Plan: Proof by Equivalence

Subject: Mathematics
Course: IB Mathematics Analysis and Approaches
Level: IB HL
Topic: Proof by Equivalence
Duration: 60 minutes

Lesson Objectives

By the end of the lesson, students will be able to:

1. Understand the concept of proof by equivalence (biconditional proof).
2. Construct and prove logical equivalence between two mathematical statements.
3. Apply proof by equivalence to various algebraic and geometric problems.

Resources

- PowerPoint presentation uploaded (as a guide for the lesson flow).
- Worksheets with exercises.

Lesson Outline

1. Introduction (10 minutes)

- **Ask**: Begin by asking students, "What does it mean for two statements to be logically equivalent? Can you think of a situation where proving both directions of an implication is necessary?"
- **Explain**: Introduce the concept of proof by equivalence, or biconditional proof, where we need to prove both directions:
  - $A \Rightarrow B$
  - $B \Rightarrow A$

  Emphasize that this proof method is used to show that two mathematical statements are logically equivalent ($A \Leftrightarrow B$).
- **Provide the Goal**: Share the learning objective: "To understand and use proof by equivalence."

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2. Guided Practice (20 minutes)

- **Example 1:** Prove that for any integer n, n^2 is divisible by 4 if and only if n is even.
- **Step 1:** Prove n is even $\Rightarrow$ n^2 is divisible by 4:
  - Let $n = 2k$, where k is an integer.
  - $n^2 = (2k)^2 = 4k^2$, which is divisible by 4.
- **Step 2:** Prove n^2 is divisible by 4 $\Rightarrow$ n is even:
  - Suppose n^2 is divisible by 4.
  - If n were odd, say $n = 2k + 1$, then $n^2 = (2k+1)^2 = 4k^2 + 4k + 1$, which is not divisible by 4.
  - Therefore, n must be even.

- **Conclusion:** n^2 is divisible by 4 $\Leftrightarrow n$ is even.
- Ask students to summarize the structure: prove both directions to complete the biconditional proof.
- **Example 2:** Prove $a + b = 0$ if and only if $a = -b$.
- **Step 1:** Prove $a + b = 0 \Rightarrow a = -b$.
  - Assume $a + b = 0$. Subtract b from both sides to get $a = -b$.
- **Step 2:** Prove $a = -b \Rightarrow a + b = 0$.
  - Assume a = −b. Substitute into the equation to get $a + b = -b + b = 0$.
- **Conclusion:** $a + b = 0 \Leftrightarrow a = -b$.
- Have students summarize how the reasoning works for proving both directions.

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3. Independent Practice (15 minutes)

Collaborative Problem-Solving

- Provide a series of problems on a worksheet, including:
  1. Linear factors.
  2. Repeated factors.
  3. Mixed factors.

Steps:

- Students attempt problems in pairs.
- **Support**: Walk around and help students who may be struggling with the concept of proving both directions. or clarify doubts.

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4. Discussion and Consolidation (10 minutes)

- **Review**: Go over the problems together, ensuring students understand how to prove both directions of an equivalence.
- **Reinforce**: Emphasize the importance of maintaining logical flow in each direction and avoiding errors like circular reasoning or oversimplifying.
- **Summary**: Recap the method for proof by equivalence: Prove both $A \Rightarrow B$ and $B \Rightarrow A$ to establish $A \Leftrightarrow B$.

5. Quick Quiz (5 minutes)

To assess student understanding, give them a brief quiz:

1. Prove $a^2 = b^2$ if and only if $a = b$ or $a = -b$.
2. Prove $x^2 + 1 = 0$ has no real solutions.
3. Prove that for any integer n, n^2 is divisible by 4 if and only if n is even.

Homework/Extension

- Finish any incomplete worksheet problems.