Proof by deduction

1. Prove that the product of two even numbers is always even.

2. Prove by deduction that if n is odd, then n^2 is also odd.

3. Prove that if a and b are real numbers such that $a + b = 0$, then $a = -b$.

4. Prove that if $x^2 - 5x + 6 = 0$, then $x = 2$ or $x = 3$.

5. Prove that the sum of three consecutive integers is divisible by 3.

6. Prove that for any integer n, $n^3 - n$ is always divisible by 6.

7. Prove that for any integer n, $n^4 - n^2$ is divisible by 12.

8. Prove that the sum of the squares of two consecutive integers is always odd.

9. Prove that if n is a positive integer, then $n^5 - n$ is divisible by 5

10. The product of three consecutive integers is increased by the middle integer. Prove that the result is a perfect cube.

Proof by deduction

ANSWERS

Proof by deduction

1. Prove that the product of two even numbers is always even.

   Let the two even numbers be 2a and 2b, where a and b are integers.

   (2a) × (2b) = 4ab = 2(2ab). Since 2ab is an integer, the product is even.

2. Prove by deduction that if n is odd, then n^2 is also odd.

   Let $n = 2k + 1$ (an odd number).

   Then: $n^2 = (2k + 1)^2 = 4k^2 + 4k + 1$

   $= 2(2k^2 + 2k) + 1$.

   Since $2k^2 + 2k$ is an integer, n^2 is odd.

3. Prove that if a and b are real numbers such that $a + b = 0$, then $a = -b$.

   We are given $a + b = 0$. To isolate a, subtract b from both sides: $a = -b$.

4. Prove that if $x^2 - 5x + 6 = 0$, then $x = 2$ or $x = 3$.

   The quadratic equation is $x^2 - 5x + 6 = 0$.

   Factorizing: $x^2 - 5x + 6 = (x - 2)(x - 3) = 0$.

   Therefore, $x = 2$ or $x = 3$

5. Prove that the sum of three consecutive integers is divisible by 3.

   Let the three consecutive integers be n, $n+1$, and $n+2$.

   $n + (n+1) + (n+2) = 3n + 3$

   $= 3(n+1)$.

   Since $n+1$ is an integer, the sum is divisible by 3

6. Prove that for any integer n, $n^3 - n$ is always divisible by 6.

   Factor $n^3 - n$: $n^3 - n = n(n^2 - 1) = n(n - 1)(n + 1)$.

   The product $n(n - 1)(n + 1)$ represents three consecutive integers, one of which is divisible by 2 and another by 3. Therefore, $n(n - 1)(n + 1)$ is divisible by 6.

7. Prove that for any integer n, $n^4 - n^2$ is divisible by 12.

   We factor $n^4 - n^2$ as $n^2(n - 1)(n + 1)$. Since $n - 1$, n, and $n + 1$ are consecutive integers, one is divisible by 2 and another by 3. At least one is divisible by 4, so the product is divisible by 12.

8. Prove that the sum of the squares of two consecutive integers is always odd.

   Let the two consecutive integers be n and $n+1$.

   Then $n^2 + (n+1)^2 = 2n^2 + 2n + 1$, which is odd since $2n^2 + 2n$ is even.

9. Prove that if n is a positive integer, then $n^5 - n$ is divisible by 5

   We factor $n^5 - n$ as $n(n^4 - 1) = n(n - 1)(n + 1)(n^2 + 1)$. Since n, $n - 1$, and $n + 1$ are consecutive integers, one is divisible by 5, so $n^5 - n$ is divisible by 5.

10. The product of three consecutive integers is increased by the middle integer. Prove

    that the result is a perfect cube.

Let the middle number be x.

$\therefore$ the product of the three consecutive integers, increased by the middle integer is

$$\begin{aligned}(x-1)x(x+1)+x &= (x^2-x)(x+1)+x \\ &= x^3+x^2-x^2-x+x \\ &= x^3 \quad \text{which is a perfect cube.}\end{aligned}$$