

## 1. Decompose:

$$\frac{5x+3}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

- Multiply through by  $(x+1)(x+2)$ :

$$5x+3 = A(x+2) + B(x+1)$$

- Expand and group terms:

$$5x+3 = Ax + 2A + Bx + B \Rightarrow (A+B)x + (2A+B)$$

- Equate coefficients:

$$A+B=5, \quad 2A+B=3$$

- Solve:  $A=-2, B=7$
- **Final Answer:**

$$\frac{5x+3}{(x+1)(x+2)} = \frac{2}{x+1} + \frac{3}{x+2}$$

## 2. Decompose:

$$\frac{7x-7}{(x-3)(x+4)} = \frac{A}{x-3} + \frac{B}{x+4}$$

- Multiply through by  $(x-3)(x+4)$ :

$$7x-7 = A(x+4) + B(x-3)$$

- Expand and group terms:

$$7x-7 = Ax + 4A + Bx - 3B \Rightarrow (A+B)x + (4A-3B)$$

- Equate coefficients:

$$A+B=7, \quad 4A-3B=-7$$

- Solve:  $A=2, B=5$ .
- **Final Answer:**

$$\frac{7x-7}{(x-3)(x+4)} = \frac{2}{x-3} + \frac{5}{x+4}$$

## 3. Decompose:

$$\frac{4x+5}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$$

- Multiply through by  $(x+2)(x-1)$ :

$$4x+5 = A(x-1) + B(x+2)$$

- Expand and group terms:

$$4x+5 = Ax - A + Bx + 2B \Rightarrow (A+B)x + (-A+2B)$$

- Equate coefficients:

$$A+B=4, \quad -A+2B=5$$

- Solve:  $A=1, B=3$ .
- **Final Answer:**

$$\frac{4x+5}{(x+2)(x-1)} = \frac{1}{x+2} + \frac{3}{x-1}$$

**Section B: Repeated Factors**

4. Decompose:

$$\frac{2x+3}{(x+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2}$$

- Multiply through by  $(x+1)^2$ :

$$2x+3 = A(x+1) + B$$

- Expand:

$$2x+3 = Ax + A + B \Rightarrow Ax + (A+B)$$

- Equate coefficients:

$$A = 2, \quad A + B = 3$$

- Solve:  $A = 2, B = 1$ .

- Final Answer:

$$\frac{2x+3}{(x+1)^2} = \frac{2}{x+1} + \frac{1}{(x+1)^2}$$

5. Decompose:

$$\frac{6x+4}{(x-2)^2} = \frac{A}{x-2} + \frac{B}{(x-2)^2}$$

- Multiply through by  $(x-2)^2$ :

$$6x+4 = A(x-2) + B$$

- Expand:

$$6x+4 = Ax - 2A + B \Rightarrow Ax + (-2A+B)$$

- Equate coefficients:

$$A = 6, \quad -2A + B = 4$$

- Solve:  $A = 6, B = 16$ .

- Final Answer:

$$\frac{6x+4}{(x-2)^2} = \frac{6}{x-2} + \frac{16}{(x-2)^2}$$

6. Decompose:

$$\frac{3x+7}{(x+3)^2} = \frac{A}{x+3} + \frac{B}{(x+3)^2}$$

- Multiply through by  $(x+3)^2$ :

$$3x+7 = A(x+3) + B$$

- Expand:

$$3x+7 = Ax + 3A + B \Rightarrow Ax + (3A+B)$$

- Equate coefficients:

$$A = 3, \quad 3A + B = 7$$

- Solve:  $A = 3, B = -2$ .

- Final Answer:

$$\frac{3x+7}{(x+3)^2} = \frac{3}{x+3} - \frac{2}{(x+3)^2}$$

**Section C: Mixed Factors (Linear and Repeated)**

7. Decompose:

$$\frac{4x^2 + 2x + 6}{(x+2)(x-1)^2} = \frac{A}{x+2} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

- Multiply through by  $(x+2)(x-1)^2$ :

$$4x^2 + 2x + 6 = A(x-1)^2 + B(x+2)(x-1) + C(x+2)$$

- Expand:

$$A(x-1)^2 = A(x^2 - 2x + 1) = Ax^2 - 2Ax + A$$

$$B(x+2)(x-1) = B(x^2 + x - 2) = Bx^2 + Bx - 2B$$

$$C(x+2) = Cx + 2C$$

Combine terms:

$$4x^2 + 2x + 6 = (A+B)x^2 + (-2A+B+C)x + (A-2B+2C)$$

- Equate coefficients:

$$A+B=4, \quad -2A+B+C=2, \quad A-2B+2C=6$$

- Solve:  $A=2, B=2, C=4$ .

- **Final Answer:**

$$\frac{4x^2 + 2x + 6}{(x+2)(x-1)^2} = \frac{2}{x+2} + \frac{2}{x-1} + \frac{4}{(x-1)^2}$$

8. Decompose:

$$\frac{2x^2 + 10x}{(x-3)(x+1)^2} = \frac{A}{x-3} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

- Multiply through by  $(x-3)(x+1)^2$ :

$$2x^2 + 10x = A(x+1)^2 + B(x-3)(x+1) + C(x-3)$$

- Expand terms and combine coefficients, following similar steps as in Question 7.

- Solving gives:  $A=3, B=-1, C=2$

- **Final Answer:**

$$\frac{2x^2 + 10x}{(x-3)(x+1)^2} = \frac{3}{x-3} - \frac{1}{x+1} + \frac{2}{(x+1)^2}$$

**Section D: Irreducible Quadratic Factors**

9. Decompose:

$$\frac{3x^2 + 4x + 2}{(x+1)(x^2 + 2x + 2)} = \frac{A}{x+1} + \frac{Bx+C}{x^2 + 2x + 2}$$

- Multiply through by  $(x+1)(x^2 + 2x + 2)$ :

$$3x^2 + 4x + 2 = A(x^2 + 2x + 2) + (Bx+C)(x+1)$$

- Expand:

$$A(x^2 + 2x + 2) = Ax^2 + 2Ax + 2A$$

$$(Bx+C)(x+1) = Bx^2 + Bx + Cx + C = Bx^2 + (B+C)x + C$$

- Equate coefficients:

$$A+B=3, \quad 2A+B+C=4, \quad 2A+C=2$$

- Solve:  $A=1, B=2, C=0$ .

- **Final Answer:**

$$\frac{3x^2 + 4x + 2}{(x+1)(x^2 + 2x + 2)} = \frac{1}{x+1} + \frac{2x}{x^2 + 2x + 2}$$

## 10. Decompose:

$$\frac{5x^2 + 7x + 3}{(x-1)(x^2 + x + 1)} = \frac{A}{x-1} + \frac{Bx + C}{x^2 + x + 1}$$

- Multiply through by  $(x-1)(x^2 + x + 1)$ :

$$5x^2 + 7x + 3 = A(x^2 + x + 1) + (Bx + C)(x-1)$$

- Expand and combine coefficients:

$$A(x^2 + x + 1) = Ax^2 + Ax + A$$

$$(Bx + C)(x-1) = Bx^2 - Bx + Cx - C = Bx^2 + (C-B)x - C$$

- Equate coefficients:

$$A + B = 5, \quad A + C - B = 7, \quad A - C = 3$$

- Solve:  $A = 5, B = 0, C = 2$

- **Final Answer:**

$$\frac{5x^2 + 7x + 3}{(x-1)(x^2 + x + 1)} = \frac{5}{x-1} + \frac{2}{x^2 + x + 1}$$

**Solutions: Challenge Section****11. Decompose:**

$$\frac{2x^2 + 3x + 1}{(x+2)^2(x-1)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

**Step 1: Multiply through by the denominator**

Multiply through by  $(x+2)^2(x-1)$ :

$$2x^2 + 3x + 1 = A(x+2)^2 + B(x+2)(x-1) + C(x-1)$$

**Step 2: Expand each term**

1. Expand  $A(x+2)^2$ :

$$A(x+2)^2 = A(x^2 + 4x + 4) = Ax^2 + 4Ax + 4A$$

2. Expand  $B(x+2)(x-1)$ :

$$B(x+2)(x-1) = B(x^2 + x - 2) = Bx^2 + Bx - 2B$$

3. Expand  $C(x-1)$ :

$$C(x-1) = Cx - C$$

Combine all terms:

$$2x^2 + 3x + 1 = (A+B)x^2 + (4A+B+C)x + (4A-2B-C)$$

**Step 3: Equate coefficients**

Equate coefficients of  $x^2$ ,  $x$ , and the constant term:

1. Coefficient of  $x^2$ :  $A + B = 2$
2. Coefficient of  $x$ :  $4A + B + C = 3$
3. Constant term:  $4A - 2B - C = 1$

**Step 4: Solve the system of equations**

1. From  $A + B = 2$ :  $B = 2 - A$ .
2. Substitute  $B = 2 - A$  into  $4A + B + C = 3$ :

$$4A + (2 - A) + C = 3 \Rightarrow 3A + C = 1 \Rightarrow C = 1 - 3A$$

3. Substitute  $B = 2 - A$  and  $C = 1 - 3A$  into  $4A - 2B - C = 1$ :

$$4A - 2(2 - A) - (1 - 3A) = 1$$

Simplify:

$$4A - 4 + 2A - 1 + 3A = 1 \Rightarrow 9A - 5 = 1 \Rightarrow 9A = 6 \Rightarrow A = \frac{2}{3}$$

4. Solve for  $B$ :

$$B = 2 - A = 2 - \frac{2}{3} = \frac{4}{3}$$

5. Solve for  $C$ :

$$C = 1 - 3A = 1 - 3\left(\frac{2}{3}\right) = 1 - 2 = -1$$

**Final Answer:**

$$\frac{2x^2 + 3x + 1}{(x+2)^2(x-1)} = \frac{\frac{2}{3}}{x-1} + \frac{\frac{4}{3}}{x+2} - \frac{1}{(x+2)^2}$$

**12. Decompose:**

$$\frac{10x^2 - 2x - 4}{(x-3)(x+1)(x^2+1)} = \frac{A}{x-3} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}$$

**Step 1: Multiply through by the denominator**

Multiply through by  $(x-3)(x+1)(x^2+1)$ :

$$10x^2 - 2x - 4 = A(x+1)(x^2+1) + B(x-3)(x^2+1) + (Cx+D)(x-3)(x+1)$$

**Step 2: Expand each term**

1. Expand  $A(x+1)(x^2+1)$ :

$$A(x+1)(x^2+1) = A(x^3 + x^2 + x + 1)$$

2. Expand  $B(x-3)(x^2+1)$ :

$$B(x-3)(x^2+1) = B(x^3 - 3x^2 + x - 3)$$

3. Expand  $(Cx+D)(x-3)(x+1)$ :

$$(Cx+D)(x-3)(x+1) = (Cx+D)(x^2-2x-3) = Cx^3 - 2Cx^2 - 3Cx + Dx^2 - 2Dx - 3D$$

Combine all terms:

$$10x^2 - 2x - 4 = (A+B+C)x^3 + (A-3B-2C+D)x^2 + (A+B-3C-2D)x + (A-3B-3D)$$

**Step 3: Equate coefficients**

Equate coefficients of  $x^3$ ,  $x^2$ ,  $x$ , and the constant term:

1. Coefficient of  $x^3$ :  $A + B + C = 0$

2. Coefficient of  $x^2$ :  $A - 3B - 2C + D = 10$

3. Coefficient of  $x$ :  $A + B - 3C - 2D = -2$

4. Constant term:  $A - 3B - 3D = -4$

**Step 4: Solve the system of equations**

1. From  $A + B + C = 0$ :  $C = -A - B$ .

2. Substitute  $C = -A - B$  into the other equations and solve the resulting system.

•  $A = 2, B = -1, C = -1, D = 3$ .

**Final Answer:**

$$\frac{10x^2 - 2x - 4}{(x-3)(x+1)(x^2+1)} = \frac{2}{x-3} - \frac{1}{x+1} - \frac{x-3}{x^2+1}$$