Name: ______________________________________ Score: _______

Teacher: ___________________________________ Date: _________

Derivatives as Rates of Change

Instructions:

Solve the following problems. Show all necessary steps. For word problems, write a brief interpretation of the solution in context.

Section A: Basic Problems

1. Find the derivative of the following functions with respect to x:

a) $f(x) = 3x^2 + 2x - 5$

b) $g(x) = 5x^3 - 4x + 7$

c) $h(x) = \left(\frac{1}{2}\right)x^2 - 3x + 4$

2. A particle moves along a straight line, and its position $s(t)$ is given by $s(t) = 4t^2 - 2t + 5$ (in meters) where t is time in seconds. Find:

a) The velocity function.

b) The velocity of the particle at $t = 3$ seconds.

3. Determine the instantaneous rate of change of the function $f(x) = 2x^2 - 3x + 1$ at $x = 2$.

Section B: Application Problems

4. A car's position is modelled by the function $s(t) = -5t^2 + 20t + 50$, where s is in meters and t is in seconds.

a) Find the average velocity between $t = 1$ and $t = 3$.

b) Find the instantaneous velocity at $t = 2$.

5. The volume of a spherical balloon is given by $V(r) = \left(\frac{4}{3}\right)\pi r^3$, where r is the radius in centimetres. Find the rate of change of the volume with respect to the radius when $r = 5$ cm.

6. The profit, $P(x)$, of a company depends on the quantity of items sold, x, and is given by

$$P(x) = 200x - 0.5x^2 - 1000.$$

a) Find the marginal profit function.

b) Determine the marginal profit when $x = 100$. Interpret your result.

Section C: Real-World Scenarios

7. A diver's height above the water is given by $s(t) = -4.9t^2 + 4.9t + 10$, where t is the time in seconds.

a) Find the average velocity between $t = 1$ and $t = 2$.

b) Find the instantaneous velocity at $t = 1$.

c) Interpret your results.

8. The volume of water in a tank is given by $V(t) = 300 + 2t - t^2$, where V is in litres and t is in seconds.

a) Find the rate of change of volume with respect to time.

b) Calculate the rate of change at $t = 3$.

c) What does a negative rate of change indicate in this context?

9. The profit function of a company is given by $P(x) = 2.3x - 0.05x^2 - 12$, where x is in thousands of tonnes.

a) Find the quantity that maximizes profit.

b) Calculate the maximum profit.

10. The temperature of an object $T(t)$, in degrees Celsius, decreases according to the function

$T(t) = 100e^{(-0.1t)}$, where t is in minutes.

a) Find the rate of change of temperature at $t = 5$ minutes.

b) Interpret the result in terms of cooling.

Section D: Higher-Order Thinking

11. A company produces widgets, and its cost function is $C(x) = 50x + 2000$, where x is the number of widgets produced.

a) Find the average cost per widget when $x = 50$.

b) Determine the marginal cost.

c) Explain how marginal cost helps in decision-making.

12. The position of a particle is given by $s(t) = t^3 - 6t^2 + 9t + 2$. Find:

a) The velocity function.

b) The acceleration function.

c) The velocity and acceleration at $t = 2$.