Derivatives as Rates of Change

Solutions

Section A: Basic Problems

1. a) $f'(x) = 6x + 2$

   b) $g'(x) = 15x^2 - 4$

   c) $h'(x) = x - 3$

2. a) $v(t) = 8t - 2$

   b) $v(3) = 22$ m/s

3. $f'(x) = 4x - 3; f'(2) = 5$

Section B: Application Problems

4. a) $v_{\text{avg}} = -15$ m/s

   b) $v(2) = 0$ m/s

5. $\frac{dV}{dr} = 4\pi r^2$; $\left.\frac{dV}{dr}\right|_{r=5} = 100\pi$ cm^2

6. a) $P'(x) = 200 - x$

   b) $P'(100) = 100$ units

Section C: Real-World Scenarios

7. a) $v_{\text{avg}} = -4.9$ m/s

   b) $v(1) = -4.9$ m/s

   c) Instantaneous velocity is consistent with the trend seen in average velocity over shorter intervals.

8. a) $\frac{dV}{dt} = 2 - 2t$

   b) $\left.\frac{dV}{dt}\right|_{t=3} = -4$ litres/s

   c) Negative rate indicates the volume is decreasing.

9. a) $x = 23$ tonnes

   b) $P(23) = 14.45$ million dollars

10. a) $\left.\frac{dT}{dt}\right|_{t=5} = -8.187$ °C/min

b) Temperature is decreasing rapidly at 5 minutes.

Section D: Higher-Order Thinking

11. a) $C_{\text{avg}} = 90$

b) $C'(x) = 50$

c) Marginal cost helps determine the cost of producing one additional unit.

12. a) $v(t) = 3t^2 - 12t + 9$

b) $a(t) = 6t - 12$

c) $v(2) = 3$; $a(2) = 0$