Second derivative test - Answers

For questions 1 - 6 answer each of the following.

a) Determine a list of possible inflection points for the function.

b) Determine the intervals on which the function is concave up and concave down.

c) Determine the inflection points of the function.

1. $f(x) = 12 + 6x^2 - x^3$

a) $x = 2$

b) Concave Up : $(-\infty,2)$

Concave Down : $(2,\infty)$

c) $x = 2$

2. $g(z) = z^4 - 12z^3 + 84z + 4$

a) z = 0 & z = 6

b) Concave Up : $(-\infty,0)\&(6,\infty)$

Concave Down : $(0,6)$

c) z = 0 & z = 6

3. $h(t) = t^4 + 12t^3 + 6t^2 - 36t + 2$

a) $t=-3\pm2\sqrt{2} = -5.8284, -0.1716$

b) Concave Up : $(-\infty,-3-2\sqrt{2})$ & $(-3+2\sqrt{2},\infty)$

Concave Down : $(-3-2\sqrt{2},-3+2\sqrt{2})$

c) $t =-3-2\sqrt{2}$ & $t=-3+2\sqrt{2}$

4. $h(w) = 8 - 5w + 2w^2 - \cos(3w)$ on $[-1,2]$

a) w = −0.6771 w = 0.6771

w = 1.4173

b) Concave Up : $(-0.6771,0.6771)$ & $(1.4173,2]$

Concave Down : $7[-1,-0.6771)$ & $(0.6771,1.4173)$

c) w = −0.6771 w = 0.6771

w = 1.4173

5. $R(z) = z(z+4)^{\frac{2}{3}}$

a) $z = -\frac{24}{5} = -4.8$ & z = −4

b) Concave Up : $(-\frac{24}{5},-4)$ & $(-4,\infty)$

Concave Down : $(-\infty,-\frac{24}{5})$

c) $z = -\frac{24}{5}$

6. $h(x) = e^{4-x^2}$

a) $x = \pm\frac{1}{\sqrt{2}} = \pm0.7071$

b) Concave Up : $(-\infty,-\frac{1}{\sqrt{2}})$ & $(\frac{1}{\sqrt{2}},\infty)$

c) Concave Down : $(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$

For problems 7 – 12 answer each of the following.

a) Identify the critical points of the function.
b) Determine the intervals on which the function increases and decreases.
c) Classify the critical points as relative maximums, relative minimums or neither.
d) Determine the intervals on which the function is concave up and concave down.
e) Determine the inflection points of the function.

7. $g(t) = t^5 - 5t^4 + 8$

a) t = 0 & t = 4
b) Increasing : $(-\infty,0)$ & $(4,\infty)$
Decreasing : $(0,4)$
c) t = 0:Relative Maximum
t =4 :Relative Minimum
d) Concave Up : $(3,\infty)$
Concave Down : $(-\infty,0)$ & $(0,3)$
e) t = 3

8. $f(x) = 5 - 8x^3 - x^4$

a) x = −6 & x = 0
b) Increasing : $(-\infty,-6)$
Decreasing : $(-6,0)$ & $(0,\infty)$
c) x = −6: Relative Maximum
x = 0: Neither
d) Concave Up : $(-4,0)$
Concave Down : $(-\infty,-4)$ & $(0,\infty)$
e) x = −4 & x = 0

9. $h(z) = z^4 - 2z^3 - 12z^2$

a) x = 0 & x = $\frac{3\pm\sqrt{105}}{4}$ = −1.8117, 3.3117
b) Increasing : $(\frac{3-\sqrt{105}}{4},0)$ & $(\frac{3+\sqrt{105}}{4},\infty)$
Decreasing : $(-\infty,\frac{3-\sqrt{105}}{4})$&$(0,\frac{3+\sqrt{105}}{4})$
c) z = $\frac{3\pm\sqrt{105}}{4}$:Relative Minimum
z = 0: Relative Maximum
z = $\frac{3\pm\sqrt{105}}{4}$: Relative Minimum
d) Concave Up : $(-\infty,-1)$ & $(2,\infty)$
Concave Down : $(-1,2)$
e) z = −1 & z = 2

10. $Q(t) = 3t - 8\sin\left(\frac{t}{2}\right)$ on $[-7,4]$

a) t = −1.4454 & t = 1.4454
b) Increasing : $[-7,-1.4454)$ & $(1.4454,4]$
Decreasing : $(-1.4454,1.4454)$
c) t=−1.4454: Relative Maximum
t=1.4454: Relative Minimum
d) Concave Up : $[-7,-6.2832)$ & $(0,4]$
Concave Down : $(-6.2832,0)$
e) t = −6.2832 & t = 0

11. $f(x) = x^{\frac{4}{3}}(x-2)$

a) x = 0 & x = $\frac{8}{7}$ = 1.1429
b) Increasing : $(-\infty,0)$ & $(\frac{8}{7},\infty)$
Decreasing : $(0,\frac{8}{7})$
c) x = 0: Relative Maximum
x = $\frac{8}{7}$: Relative Minimum
d) Concave Up : $(\frac{2}{7},\infty)$
Concave Down : $(-\infty,0)$ & $(0,\frac{2}{7})$
e) x = $\frac{2}{7}$

12. $P(w) = we^{4w}$

a) w = $-\frac{1}{4}$
b) Increasing : $(-\frac{1}{4},\infty)$ Decreasing : $(-\infty,-\frac{1}{4})$
c) w=$-\frac{1}{4}$: Relative Minimum
d) Concave Up : $(-\frac{1}{2},\infty)$
Concave Down : $(-\infty,-\frac{1}{2})$
e) w=$-\frac{1}{2}$