Intermediate value theorem - Answers

For questions 1 - 6, use the Intermediate Value Theorem to show that the given equation has at least one solution in the indicated interval. Note that you are NOT asked to find the solution only show that at least one must exist in the indicated interval.

1. $25 - 8x^2 - x^3 = 0$ on $[-2, 4]$

f(-2)=1 f(4)=−167
Therefore, we have,
f(4) = −167 < 0 < 1 = f(-2)
So, by the Intermediate Value Theorem There must be a number c such that,
−2< c <4 & f(c)=0

2. $w^2 - 4\ln(5w + 2) = 0$ on $[0, 4]$

f(0)= −2.7726 f(4)= 3.6358
Therefore, we have,
f(0) = −2.7726 < 0 < 3.6358 = f(4)
So, by the Intermediate Value Theorem there must be a number c such that,
0 < c < 4 & f(c)=0

3. $4t + 10\mathbf{e}^t - \mathbf{e}^{2t} = 0$ on $[1, 3]$

f(1)=23.7938 f(3)=−190.5734
Therefore, we have,
f(3) = −190.5734 < 0 < 23.7938 = f(1)
So, by the Intermediate Value Theorem there must be a number c such that,
1 < c < 3 & f(c)=0

4. $f(x)=\frac{1}{16}x^4 - x^3 + 3;\ [1,2]$

f(1)=2.0625 f(2)=−4
Therefore, we have,
f(2) = −4 < 0 < 2.0625 = f(1)
So, by the Intermediate Value Theorem there must be a number c such that,

1 < c < 2 & f(c)=0

5. $f(x)=x^3 + 3x - 2;\ [0,1]$

f(0) = −2 f(1)= 2
Therefore, we have,
f(0) = −2 < 0 < 2 = f(1)
So, by the Intermediate Value Theorem there must be a number c such that,

0 < c < 1 & f(c)=0

6. $f(x)=x^2 - x - \cos x;\ [0,\pi]$

f(0) = −1 f(π)= 7.728
Therefore, we have,
f(0) = −1 < 0 < 7.728 = f(π)
So, by the Intermediate Value Theorem there must be a number c such that,

$0 < c < 1$ & $f(c)=0$

In 7- 9, verify that the Intermediate Value Theorem guarantees that there is a zero in the interval [0,1] for the given function. Use a graphing calculator to find the zero.

7. $f(x)=x^3+x-1$

$(0) = -1$ $f(1)= 1$
Therefore, we have,
$f(0) = -1 < 0 < 1 = f(1)$
So, by the Intermediate Value Theorem there must be a number c such that,
$0 < c < 1$ & $f(c)=0$

$X = 0.682$

8. $f(x)=x^3+3x-2$

$f(0) = -2$ $f(1)= 2$
Therefore, we have,
$f(0) = -2 < 0 < 2 = f(1)$
So, by the Intermediate Value Theorem there must be a number c such that,
$0 < c < 1$ & $f(c)=0$

$X = 0.596$

9. $g(t)=2\cos t-3t$

$f(0) = 2$ $f(1)= -1.919$
Therefore, we have,
$f(1) = -1.919 < 0 < 2 = f(0)$
So, by the Intermediate Value Theorem there must be a number c such that,
$0 < c < 1$ & $f(c)=0$

$X = 0.564$

In questions 10 – 12, verify that the Intermediate Value Theorem applies to the indicated interval and find the value of c guaranteed by the theorem.

10. $f(x)=x^2+x-1,\quad [0,5],\qquad f(c)=11$

$f(0) = -1$ $f(5)= 29$
Therefore, we have,
$f(0) = -1 < 0 < 29 = f(5)$
So, by the Intermediate Value Theorem there must be a number c such that,
$0 < c < 5$ & $f(c)=11$

$c = 3$

11. $f(x)=x^2-6x+8,\quad [0,3],\quad f(c)=0$

$f(0) = 8$ $f(3)= -1$
Therefore, we have,
$f(3) = -1 < 0 < 8 = f(0)$
So, by the Intermediate Value Theorem there must be a number c such that,

$0 < c < 3$ & $f(c)=0$

$c = 4$

12. $f(x)=\dfrac{x^2+x}{x-1}, \quad \left[\dfrac{5}{2},4\right], \quad f(c)=6$

$f(5/2) = 35/6 \quad f(4)= 20/3$

Therefore, we have,

$f(3) = -1 < 0 < 8 = f(0)$

So, by the Intermediate Value Theorem there must be a number c such that,

$5/2 < c < 4$ & $f(c)=6$

$c = 3$