Multiplication of matrices

Student name: _____ Score: _____

1. Let $\boldsymbol{A}=\begin{pmatrix}1 & x & -1\\ 3 & 1 & 4\end{pmatrix}$ and $\boldsymbol{B}=\begin{pmatrix}3\\ x\\ 2\end{pmatrix}$.

(a) Find $\boldsymbol{AB}$.

(b) The matrix $\boldsymbol{C}=\begin{pmatrix}20\\ 28\end{pmatrix}$ and $2\boldsymbol{AB}=\boldsymbol{C}$. Find the value of x.

2. Let $\boldsymbol{A}=\begin{pmatrix}1 & 2\\ 3 & -1\end{pmatrix}$ and $\boldsymbol{B}=\begin{pmatrix}3 & 0\\ -2 & 1\end{pmatrix}$.

Find $\boldsymbol{AB}$.

3. Let $\boldsymbol{A}=\begin{pmatrix}5 & 1\\ 6 & 2\end{pmatrix}$ and $\boldsymbol{B}=\begin{pmatrix}2 & -1\\ -6 & 5\end{pmatrix}$.

Find $\boldsymbol{AB}$.

4. Let $\boldsymbol{W}=\begin{pmatrix}1 & 3 & 2\\ 2 & 0 & 1\\ 0 & 1 & 3\end{pmatrix}$ and $\boldsymbol{P}=\begin{pmatrix}2\\ 3\\ 1\end{pmatrix}$.

(a) Find $\boldsymbol{WP}$.

(b) Given that $2\boldsymbol{WP}+\boldsymbol{S}=\begin{pmatrix}26\\ 12\\ 10\end{pmatrix}$, find $\boldsymbol{S}$.

5. Let $\boldsymbol{A}=\begin{pmatrix}1 & -2\\ 3 & 4\end{pmatrix}$ and $\boldsymbol{B}=\begin{pmatrix}-5\\ 5\end{pmatrix}$.

Find $\boldsymbol{AB}$.

6. Let $\boldsymbol{A}=\begin{pmatrix}1 & 2\\ 3 & 0\end{pmatrix}$, $\boldsymbol{B}=\begin{pmatrix}2 & 1\\ 1 & q\end{pmatrix}$, $\boldsymbol{C}=\begin{pmatrix}p & -1\\ 6 & 3\end{pmatrix}$, so that $\boldsymbol{AB}=\boldsymbol{C}$.

(a) Find the value of p.

(b) Find the value of q.

7. Let A be a 2×2 matrix and B an $m\times n$ matrix, where $A=\begin{pmatrix}-2 & 0\\ 1 & 3\end{pmatrix}$ and $B=\begin{pmatrix}1 & 0 & 3\\ -1 & 3 & 0\end{pmatrix}$.

(a) Find AB.

(b) Let C be a $p\times4$ matrix. Given that the product BC exists, write down the value of p.

8. Let $P=\begin{pmatrix}3 & 1\\ 5 & 2\end{pmatrix}$ and $Q=\begin{pmatrix}4 & -2\\ -10 & 6\end{pmatrix}$. Find PQ.

9. Let $A=\begin{pmatrix}0 & 3\\ -2 & 4\end{pmatrix}$ and $B=\begin{pmatrix}-4 & 0\\ 5 & 1\end{pmatrix}$.

(a) Find AB.

(b) Given that $X-2A=B$, find X.

10. Let $A=\begin{pmatrix}1 & 2\\ 3 & 0\end{pmatrix}$, $B=\begin{pmatrix}2 & 1\\ 1 & 3\end{pmatrix}$ Find AB.

11. Let $C=\begin{pmatrix}-2 & 4\\ 1 & 7\end{pmatrix}$ and $D=\begin{pmatrix}5 & 2\\ -1 & a\end{pmatrix}$. Find CD.

12. Let $M=\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}$.

(a) (i) Find M^2.

(ii) Show that $M^3=\begin{pmatrix}1 & 6\\ 0 & 1\end{pmatrix}$.

It may now be assumed that $M^n=\begin{pmatrix}1 & 2n\\ 0 & 1\end{pmatrix}$, for $n\geq4$. The sum T_n is defined by

$$T_n=M^1+M^2+M^3+\ldots+M^n.$$

(b) (i) Write down M^4.

(ii) Find T_4.

(c) Find T_{100}

Multiplication of matrices

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1. Let $\boldsymbol{A}=\begin{pmatrix}1 & x & -1\\ 3 & 1 & 4\end{pmatrix}$ and $\boldsymbol{B}=\begin{pmatrix}3\\ x\\ 2\end{pmatrix}$.

(a) Find $\boldsymbol{AB}$. $\begin{pmatrix}1+x^2\\ 17+x\end{pmatrix}$

(b) The matrix $\boldsymbol{C}=\begin{pmatrix}20\\ 28\end{pmatrix}$ and $2\boldsymbol{AB}=\boldsymbol{C}$. Find the value of x. $x = -3$

2. Let $\boldsymbol{A}=\begin{pmatrix}1 & 2\\ 3 & -1\end{pmatrix}$ and $\boldsymbol{B}=\begin{pmatrix}3 & 0\\ -2 & 1\end{pmatrix}$.

Find $\boldsymbol{AB}$. $\begin{pmatrix}-1 & 2\\ 11 & -1\end{pmatrix}$

3. Let $\boldsymbol{A}=\begin{pmatrix}5 & 1\\ 6 & 2\end{pmatrix}$ and $\boldsymbol{B}=\begin{pmatrix}2 & -1\\ -6 & 5\end{pmatrix}$.

Find $\boldsymbol{AB}$. $\begin{pmatrix}4 & 0\\ 0 & 4\end{pmatrix}$

4. Let $\boldsymbol{W}=\begin{pmatrix}1 & 3 & 2\\ 2 & 0 & 1\\ 0 & 1 & 3\end{pmatrix}$ and $\boldsymbol{P}=\begin{pmatrix}2\\ 3\\ 1\end{pmatrix}$.

(a) Find $\boldsymbol{WP}$. $\begin{pmatrix}13\\ 5\\ 6\end{pmatrix}$

(b) Given that $2\boldsymbol{WP}+\boldsymbol{S}=\begin{pmatrix}26\\ 12\\ 10\end{pmatrix}$, find $\boldsymbol{S}$. $\begin{pmatrix}0\\ 2\\ -2\end{pmatrix}$

5. Let $\boldsymbol{A}=\begin{pmatrix}1 & -2\\ 3 & 4\end{pmatrix}$ and $\boldsymbol{B}=\begin{pmatrix}-5\\ 5\end{pmatrix}$.

Find $\boldsymbol{AB}$. $\begin{pmatrix}-15\\ 5\end{pmatrix}$

6. Let $\boldsymbol{A}=\begin{pmatrix}1 & 2\\ 3 & 0\end{pmatrix}$, $\boldsymbol{B}=\begin{pmatrix}2 & 1\\ 1 & q\end{pmatrix}$, $\boldsymbol{C}=\begin{pmatrix}p & -1\\ 6 & 3\end{pmatrix}$, so that $\boldsymbol{AB}=\boldsymbol{C}$.

(a) Find the value of p. $p = 4$

(b) Find the value of q. $q = -1$

7. Let A be a 2×2 matrix and B an $m\times n$ matrix, where $A=\begin{pmatrix}-2 & 0\\ 1 & 3\end{pmatrix}$ and $B=\begin{pmatrix}1 & 0 & 3\\ -1 & 3 & 0\end{pmatrix}$.

(a) Find AB. $\begin{pmatrix}-2 & 0 & -6\\ -2 & 9 & 3\end{pmatrix}$

(b) Let C be a $p\times4$ matrix. Given that the product BC exists, write down the value of p. $p=3$

8. Let $P=\begin{pmatrix}3 & 1\\ 5 & 2\end{pmatrix}$ and $Q=\begin{pmatrix}4 & -2\\ -10 & 6\end{pmatrix}$. Find PQ. $\begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix}$

9. Let $A=\begin{pmatrix}0 & 3\\ -2 & 4\end{pmatrix}$ and $B=\begin{pmatrix}-4 & 0\\ 5 & 1\end{pmatrix}$.

(a) Find AB. $\begin{pmatrix}15 & 3\\ 28 & 4\end{pmatrix}$

(b) Given that $X-2A=B$, find X. $\begin{pmatrix}-4 & 6\\ 1 & 9\end{pmatrix}$

10. Let $A=\begin{pmatrix}1 & 2\\ 3 & 0\end{pmatrix}$, $B=\begin{pmatrix}2 & 1\\ 1 & 3\end{pmatrix}$ Find AB. $\begin{pmatrix}4 & 7\\ 6 & 3\end{pmatrix}$

11. Let $C=\begin{pmatrix}-2 & 4\\ 1 & 7\end{pmatrix}$ and $D=\begin{pmatrix}5 & 2\\ -1 & a\end{pmatrix}$. Find CD. $\begin{pmatrix}-14 & -4+4a\\ -2 & 2+7a\end{pmatrix}$

12. Let $M=\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}$.

(a) (i) Find M^2. $\begin{pmatrix}1 & 4\\ 0 & 1\end{pmatrix}$

(ii) Show that $M^3=\begin{pmatrix}1 & 6\\ 0 & 1\end{pmatrix}$.

It may now be assumed that $M^n=\begin{pmatrix}1 & 2n\\ 0 & 1\end{pmatrix}$, for $n\geq4$. The sum T_n is defined by

$$T_n=M^1+M^2+M^3+\ldots+M^n.$$

(b) (i) Write down M^4. $\begin{pmatrix}1 & 8\\ 0 & 1\end{pmatrix}$

(ii) Find T_4. $\begin{pmatrix}4 & 20\\ 0 & 4\end{pmatrix}$

(c) Find T_{100} $\begin{pmatrix}100 & 10100\\ 0 & 100\end{pmatrix}$