



10.3 – Binomial expansion

Student name: _____ Score: _____

1. (a) Find the binomial series expansion of

$$\sqrt{4 - 9x}, \quad |x| < \frac{4}{9}$$

in ascending powers of x , up to and including the term in x^2
Give each coefficient in its simplest form.

- (b) Use the expansion from part (a), with a suitable value of x , to find an approximate value for $\sqrt{310}$

Show all your working and give your answer to 3 decimal places.

2. $f(x) = (2 + kx)^{-3}, \quad |kx| < 2$, where k is a positive constant

The binomial expansion of $f(x)$, in ascending powers of x , up to and including the term in x^2 is

$$A + Bx + \frac{243}{16}x^2$$

where A and B are constants.

- (a) Write down the value of A .
(b) Find the value of k .
(c) Find the value of B .

3. Use the binomial series to find the expansion of

$$\frac{1}{(2 + 5x)^3}, \quad |x| < \frac{2}{5}$$

in ascending powers of x , up to and including the term in x^3 .
Give each coefficient as a fraction in its simplest form.

4. (a) Find the binomial expansion of

$$(4 + 5x)^{\frac{1}{2}}, \quad |x| < \frac{4}{5}$$

in ascending powers of x , up to and including the term in x^2 .
Give each coefficient in its simplest form.

- (b) Find the exact value of $(4 + 5x)^{\frac{1}{2}}$ when $x = \frac{1}{10}$

Give your answer in the form $k\sqrt{2}$, where k is a constant to be determined.

- (c) Substitute $x = \frac{1}{10}$ into your binomial expansion from part (a) and hence find an approximate value for $\sqrt{2}$

Give your answer in the form $\frac{p}{q}$ where p and q are integers.

5. Given that the binomial expansion of $(1 + kx)^{-4}$, $|kx| < 1$, is

$$1 - 6x + Ax^2 + \dots$$

- (a) find the value of the constant k ,
(b) find the value of the constant A , giving your answer in its simplest form.

6. Given

$$f(x) = (2 + 3x)^{-3}, \quad |x| < \frac{2}{3}$$

find the binomial expansion of $f(x)$, in ascending powers of x , up to and including the term in x^3 .

Give each coefficient as a simplified fraction.

7.
$$f(x) = \frac{6}{\sqrt{9 - 4x}}, \quad |x| < \frac{9}{4}$$

- (a) Find the binomial expansion of $f(x)$ in ascending powers of x , up to and including the term in x^3 . Give each coefficient in its simplest form.

Use your answer to part (a) to find the binomial expansion in ascending powers of x , up to and including the term in x^3 , of

(b) $g(x) = \frac{6}{\sqrt{9+4x}}, \quad |x| < \frac{9}{4}$

(c) $h(x) = \frac{6}{\sqrt{9-8x}}, \quad |x| < \frac{9}{8}$



8. (a) Expand

$$\frac{1}{(2-5x)^2}, \quad |x| < \frac{2}{5}$$

in ascending powers of x , up to and including the term in x^2 , giving each term as a simplified fraction.

Given that the binomial expansion of $\frac{2+kx}{(2-5x)^2}$, $|x| < \frac{2}{5}$, is

$$\frac{1}{2} + \frac{7}{4}x + Ax^2 + \dots$$

- (b) find the value of the constant k ,
(c) find the value of the constant A .

9.
$$f(x) = \frac{1}{\sqrt{(9+4x^2)}}, \quad |x| < \frac{3}{2}$$

Find the first three non-zero terms of the binomial expansion of $f(x)$ in ascending powers of x . Give each coefficient as a simplified fraction.

(a) Use the binomial theorem to expand

$$(2-3x)^{-2}, \quad |x| < \frac{2}{3},$$

10. in ascending powers of x , up to and including the term in x^3 . Give each coefficient as a simplified fraction.

$$f(x) = \frac{a+bx}{(2-3x)^2}, \quad |x| < \frac{2}{3}, \quad \text{where } a \text{ and } b \text{ are constants.}$$

In the binomial expansion of $f(x)$, in ascending powers of x , the coefficient of x is 0 and the coefficient of x^2 is $\frac{9}{16}$. Find

- (b) the value of a and the value of b ,
(c) the coefficient of x^3 , giving your answer as a simplified fraction.

11. (a) Find the binomial expansion of

$$\sqrt{(1-8x)}, \quad |x| < \frac{1}{8},$$

in ascending powers of x up to and including the term in x^3 , simplifying each term.

(b) Show that, when $x = \frac{1}{100}$, the exact value of $\sqrt{(1-8x)}$ is $\frac{\sqrt{23}}{5}$.

(c) Substitute $x = \frac{1}{100}$ into the binomial expansion in part (a) and hence obtain an approximation to $\sqrt{23}$. Give your answer to 5 decimal places.

12.
$$f(x) = \frac{1}{\sqrt{4+x}}, \quad |x| < 4$$

Find the binomial expansion of $f(x)$ in ascending powers of x , up to and including the term in x^3 . Give each coefficient as a simplified fraction.

13. (a) Expand $\frac{1}{\sqrt{4-3x}}$, where $|x| < \frac{4}{3}$, in ascending powers of x up to and including the term in x^2 . Simplify each term.

(b) Hence, or otherwise, find the first 3 terms in the expansion of $\frac{x+8}{\sqrt{4-3x}}$ as a series in ascending powers of x .

