10.3 – Binomial expansion

Student name: ________________________________ Score: _______

1. (a)

$\sqrt{(4-9x)} = (4-9x)^{\frac{1}{2}} = \underline{(4)^{\frac{1}{2}}}\left(1-\frac{9x}{4}\right)^{\frac{1}{2}} = \underline{2}\left(1-\frac{9x}{4}\right)^{\frac{1}{2}}$
$=\{2\}\left[1+\left(\frac{1}{2}\right)(kx)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2+\ldots\right]$
$=\{2\}\left[1+\left(\frac{1}{2}\right)\left(-\frac{9x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(-\frac{9x}{4}\right)^2+\ldots\right]$
$=2\left[1-\frac{9}{8}x-\frac{81}{128}x^2+\ldots\right]$
$=2-\frac{9}{4}x;-\frac{81}{64}x^2+\ldots$

(b) $\sqrt{310} = 10\sqrt{3.1} = 10\sqrt{(4-9(0.1))}$, so $x = 0.1$

When $x=0.1$ $\sqrt{(4-9x)} \approx 2-\frac{9}{4}(0.1) - \frac{81}{64}(0.1)^2+\ldots$

$= 2-0.225-0.01265625 = 1.76234375$

So, $\sqrt{310} \approx 17.6234375 = \underline{17.623}$ (3 dp)

2. (a) $\left\{(2+kx)^{-3} = 2^{-3}\left(1+\frac{kx}{2}\right)^{-3} = \frac{1}{8}\left(1+(-3)\left(\frac{kx}{2}\right)+\frac{(-3)(-3-1)}{2!}\left(\frac{kx}{2}\right)^2+\ldots\right)\right\}, k>0$

$\{A=\}\ \frac{1}{8}$

(b) $\left(\frac{1}{8}\right)\frac{(-3)(-4)}{2!}\left(\frac{k}{2}\right)^2$

$\left\{\text{So, } \left(\frac{1}{8}\right)\frac{(-3)(-4)}{2!}\left(\frac{k}{2}\right)^2 = \frac{243}{16} \Rightarrow \frac{3}{16}k^2 = \frac{243}{16} \Rightarrow k^2 = 81\right\}$

So, $k=9$

(c) "$\left(\frac{1}{8}\right)$" $(-3)\left(\frac{k}{2}\right)$

$\left\{\text{So, } B = \left(\frac{1}{8}\right)(-3)\left(\frac{9}{2}\right) \Rightarrow\right\}\ \underline{B=-\frac{27}{16}}$

3.

$$\left\{\frac{1}{(2+5x)^3} = \right\}(2+5x)^{-3}$$

$$= \underline{(2)^{-3}}\left(1+\frac{5x}{2}\right)^{-3} \quad = \underline{\frac{1}{8}}\left(1+\frac{5x}{2}\right)^{-3}$$

$$= \left\{\frac{1}{8}\right\}\left[1+(-3)(kx)+\frac{(-3)(-4)}{2!}(kx)^2+\frac{(-3)(-4)(-5)}{3!}(kx)^3+\ldots\right]$$

$$= \left\{\frac{1}{8}\right\}\left[1+(-3)\left(\frac{5x}{2}\right)+\frac{(-3)(-4)}{2!}\left(\frac{5x}{2}\right)^2+\frac{(-3)(-4)(-5)}{3!}\left(\frac{5x}{2}\right)^3+\ldots\right]$$

$$= \frac{1}{8}\left[1-\frac{15}{2}x+\frac{75}{2}x^2-\frac{625}{4}x^3+\ldots\right]$$

$$= \frac{1}{8}\left[1-7.5x+37.5x^2-156.25x^3+\ldots\right]$$

$$= \frac{1}{8}-\frac{15}{16}x;+\frac{75}{16}x^2-\frac{625}{32}x^3+\ldots$$

or $\frac{1}{8}-\frac{15}{16}x;+4\frac{11}{16}x^2-19\frac{17}{32}x^3+\ldots$

4. (a)

$$(4+5x)^{\frac{1}{2}} = \underline{(4)^{\frac{1}{2}}}\left(1+\frac{5x}{4}\right)^{\frac{1}{2}} = \underline{2}\left(1+\frac{5x}{4}\right)^{\frac{1}{2}}$$

$$= \{2\}\left[1+\left(\frac{1}{2}\right)(kx)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2+\ldots\right]$$

$$= \{2\}\left[1+\left(\frac{1}{2}\right)\left(\frac{5x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(\frac{5x}{4}\right)^2+\ldots\right]$$

$$= 2\left[1+\frac{5}{8}x-\frac{25}{128}x^2+\ldots\right]$$

$$= 2+\frac{5}{4}x;-\frac{25}{64}x^2+\ldots$$

(b) $\left\{x=\frac{1}{10} \Rightarrow (4+5(0.1))^{\frac{1}{2}} = \sqrt{4.5} = \sqrt{\frac{9}{2}} = \frac{3}{\underline{\underline{\sqrt{2}}}} = \frac{3}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}}\right\}$

$= \frac{3}{2}\sqrt{2}$

(c) $\frac{3}{2}\sqrt{2}$ or $1.5\sqrt{2}$ or $\frac{3}{\underline{\underline{\sqrt{2}}}} = 2+\frac{5}{4}\left(\frac{1}{10}\right)-\frac{25}{64}\left(\frac{1}{10}\right)^2+\ldots \quad \{=2.121\ldots\}$

So, $\frac{3}{2}\sqrt{2} = \frac{543}{256}$ or $\frac{3}{\underline{\underline{\sqrt{2}}}} = \frac{543}{256}$

$\sqrt{2} = \frac{181}{128}$ or $\sqrt{2} = \frac{256}{181}$

5. (a) $\left\{(1+kx)^{-4} = 1+(-4)(kx)+\frac{(-4)(-4-1)}{2!}(kx)^2+\dots\right\}$

Either $(-4)k=-6$ **or** $(1+kx)^{-4}=1+(-4)(kx)$

leading to $k=\frac{3}{2}$

(b) $\frac{(-4)(-5)}{2}(k)^2$

$\left\{A=\frac{(-4)(-5)}{2!}\left(\frac{3}{2}\right)^2\right\} \Rightarrow A=\frac{45}{2}$

6. $(2+3x)^{-3}=\underline{(2)^{-3}}\left(1+\frac{3x}{2}\right)^{-3} = \underline{\frac{1}{8}}\left(1+\frac{3x}{2}\right)^{-3}$

$=\left\{\frac{1}{8}\right\}\left[1+(-3)(kx)+\frac{(-3)(-4)}{2!}(kx)^2+\frac{(-3)(-4)(-5)}{3!}(kx)^3+\dots\right]$

$=\left\{\frac{1}{8}\right\}\left[\underline{1+(-3)\left(\frac{3x}{2}\right)+\frac{(-3)(-4)}{2!}\left(\frac{3x}{2}\right)^2+\frac{(-3)(-4)(-5)}{3!}\left(\frac{3x}{2}\right)^3+\dots}\right]$

$=\frac{1}{8}\left[1-\frac{9}{2}x;+\frac{27}{2}x^2-\frac{135}{4}x^3+\dots\right]$

$=\frac{1}{8}-\frac{9}{16}x;+\frac{27}{16}x^2-\frac{135}{32}x^3+\dots$

7. (a) $=\dots\left(1+\left(-\tfrac{1}{2}\right)(kx);+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(kx)^2+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}(kx)^3+\dots\right)$

$=2\left(1+\frac{2}{9}x+\dots\right)$

$=2+\frac{4}{9}x+\frac{4}{27}x^2+\frac{40}{729}x^3+\dots$

(b) $\mathrm{g}(x)=2-\frac{4}{9}x+\frac{4}{27}x^2-\frac{40}{729}x^3+\dots$

(c) $\mathrm{h}(x)=2+\frac{4}{9}(2x)+\frac{4}{27}(2x)^2+\frac{40}{729}(2x)^3+\dots$

$\left(=2+\frac{8}{9}x+\frac{16}{27}x^2+\frac{320}{729}x^3+\dots\right)$

8. (a) $\dfrac{1}{(2-5x)^2} = (2-5x)^{-2} = \underline{(2)^{-2}}\left(1-\dfrac{5x}{2}\right)^{-2} = \underline{\dfrac{1}{4}}\left(1-\dfrac{5x}{2}\right)^{-2}$

$$= \left\{\frac{1}{4}\right\}\left[\ 1+(-2)(**x)+\frac{(-2)(-3)}{2!}(**x)^2+\dots\ \right]$$

$$= \left\{\frac{1}{4}\right\}\left[\ \underline{1+(-2)\left(-\frac{5x}{2}\right)+\frac{(-2)(-3)}{2!}\left(-\frac{5x}{2}\right)^2+\dots}\ \right]$$

$$= \frac{1}{4}\left[\ 1+5x;+\frac{75}{4}x^2+\dots\ \right]$$

$$= \frac{1}{4}+\frac{5}{4}x;+\frac{75}{16}x^2+\dots$$

(b) $\left\{\dfrac{2+kx}{(2-5x)^2}\right\} = (2+kx)\left(\dfrac{1}{4}+\dfrac{5}{4}x+\left\{\dfrac{75}{16}x^2+\dots\right\}\right)$

x terms: $\dfrac{2(5x)}{4}+\dfrac{kx}{4}=\dfrac{7x}{4}$

giving, $10+k=7 \Rightarrow \underline{k=-3}$

(c) x^2 terms: $\dfrac{150x^2}{16}+\dfrac{5kx^2}{4}$

So, $A = \dfrac{75}{8}+\dfrac{5(-3)}{4} = \dfrac{75}{8}-\dfrac{15}{4} = \dfrac{45}{8}$

9. $f(x)=(\ \dots\ +\ \dots)^{-\frac{1}{2}}$

$$=9^{-\frac{1}{2}}(\ \dots\ +\ \dots)^{\dots}$$

$$\left(1+kx^2\right)^n = 1+nkx^2+\dots$$

$$\left(1+kx^2\right)^{-\frac{1}{2}} = \dots + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\left(kx^2\right)^2$$

$$\left(1+\frac{4}{9}x^2\right)^{-\frac{1}{2}} = 1-\frac{2}{9}x^2+\frac{2}{27}x^4$$

$$f(x)=\frac{1}{3}-\frac{2}{27}x^2+\frac{2}{81}x^4$$

10. (a) $(2-3x)^{-2} = 2^{-2}\left(1-\frac{3}{2}x\right)^{-2}$

$$\left(1-\frac{3}{2}x\right)^{-2} = 1+(-2)\left(-\frac{3}{2}x\right)+\frac{-2.-3}{1.2}\left(-\frac{3}{2}x\right)^2+\frac{-2.-3.-4}{1.2.3}\left(-\frac{3}{2}x\right)^3+\ \dots$$

$$= 1+3x+\frac{27}{4}x^2+\frac{27}{2}x^3+\ \dots$$

$$(2-3x)^{-2} = \frac{1}{4}+\frac{3}{4}x+\frac{27}{16}x^2+\frac{27}{8}x^3+\ \dots$$

(b) $f(x) = (a+bx)\left(\frac{1}{4}+\frac{3}{4}x+\frac{27}{16}x^2+\frac{27}{8}x^3+\ \dots\right)$

Coefficient of x; $\quad \frac{3a}{4}+\frac{b}{4}=0 \quad (3a+b=0)$

Coefficient of x^2; $\quad \frac{27a}{16}+\frac{3b}{4}=\frac{9}{16} \quad (9a+4b=3)$

Leading to $\quad a=-1,\ b=3$

(c) Coefficient of x^3 is $\frac{27a}{8}+\frac{27b}{16}=\frac{27}{8}\times(-1)+\frac{27}{16}\times 3$

$$=\frac{27}{16}$$

11. (a) $(1-8x)^{\frac{1}{2}} = 1+\left(\tfrac{1}{2}\right)(-8x)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}(-8x)^2+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}(-8x)^3+\dots$

$$=1-4x-8x^2;-32x^3-\ \dots$$

(b) $\sqrt{(1-8x)} = \sqrt{\left(1-\frac{8}{100}\right)}$

$$=\sqrt{\frac{92}{100}}=\sqrt{\frac{23}{25}}=\frac{\sqrt{23}}{5} \quad *$$

(c) $1-4x-8x^2-32x^3 = 1-4(0.01)-8(0.01)^2-32(0.01)^3$

$$=1-0.04-0.0008-0.000\,032=0.959\,168$$

$$\sqrt{23}=5\times 0.959\,168$$

$$=4.795\,84$$

12. $f(x)=\frac{1}{\sqrt{(4+x)}}=(4+x)^{-\frac{1}{2}}$

$=(4)^{-\frac{1}{2}}(1+\ \dots\ )^{\dots} \qquad \frac{1}{2}(1+\ \dots\ )^{\dots}$ or $\frac{1}{2\sqrt{(1+\dots)}}$

$$=\ \dots\left(1+\left(-\tfrac{1}{2}\right)\left(\frac{x}{4}\right)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\left(\frac{x}{4}\right)^2+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}\left(\frac{x}{4}\right)^3+\ \dots\right)$$

$$=\frac{1}{2}-\frac{1}{16}x,+\frac{3}{256}x^2-\frac{5}{2048}x^3+\ \dots$$

13. (a) $\dfrac{1}{\sqrt{(4-3x)}} = (4-3x)^{-\frac{1}{2}} = \underline{(4)^{-\frac{1}{2}}}\left(1-\dfrac{3x}{4}\right)^{-\frac{1}{2}} = \dfrac{1}{\underline{2}}\left(1-\dfrac{3x}{4}\right)^{-\frac{1}{2}}$

$$=\tfrac{1}{2}\left[\ \underline{1+(-\tfrac{1}{2})(**x);+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(**x)^2+\ldots}\ \right]$$

$$=\frac{1}{2}\left[\ \underline{1+(-\tfrac{1}{2})(-\tfrac{3x}{4})+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-\tfrac{3x}{4})^2+\ldots}\ \right]$$

$$=\tfrac{1}{2}\left[\ 1+\tfrac{3}{8}x;+\ \tfrac{27}{128}x^2+\ldots\ \right]$$

$$\left\{=\frac{1}{2}+\frac{3}{16}x;+\ \frac{27}{256}x^2+\ldots\right\}$$

(b) $(x+8)\left(\dfrac{1}{2}+\dfrac{3}{16}x\ +\ \dfrac{27}{256}x^2+\ldots\right)$

$$= \qquad \tfrac{1}{2}x+\tfrac{3}{16}x^2+\ldots..$$

$$+\ 4+\tfrac{3}{2}x+\tfrac{27}{32}x^2+\ldots..$$

$$=\ 4+2x;+\frac{33}{32}x^2+\ldots$$