Integration

Student name: ________________________________ Score: _______

1. Let $f(x) = \sqrt[3]{x^4} - \frac{1}{2}$.

Find $\int f(x)\mathrm{d}x$.

2. Let $f'(x) = 12x^2 - 2$.

Given that $f(-1) = 1$, find $f(x)$.

3. The curve $y = f(x)$ passes through the point (2, 6).

Given that $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 5$, find y in terms of x.

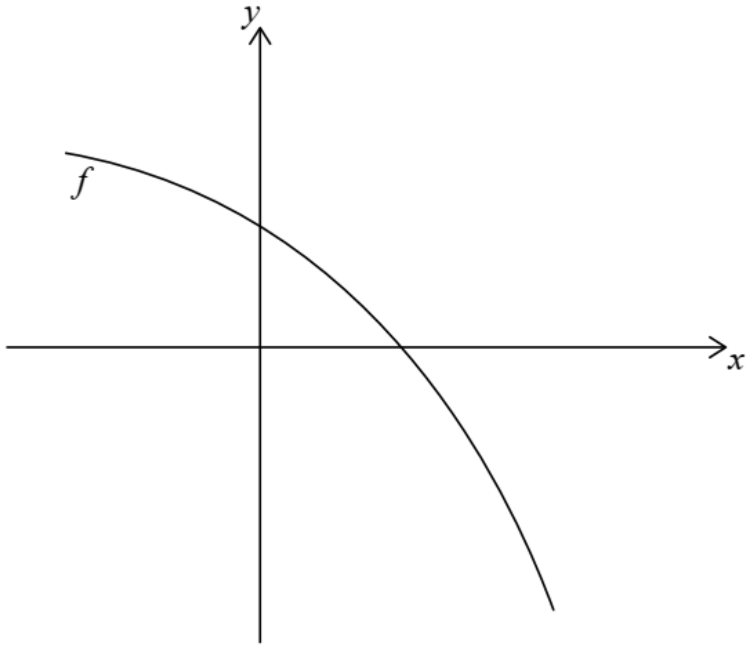
4. Let $f(x) = (3x+4)^5$. Find

$\int f(x)\mathrm{d}x$.

5. (a) Find $\int \frac{1}{2x+3}\mathrm{d}x$.

(b) Given that $\int_0^3 \frac{1}{2x+3}\mathrm{d}x = \ln\sqrt{P}$, find the value of P.

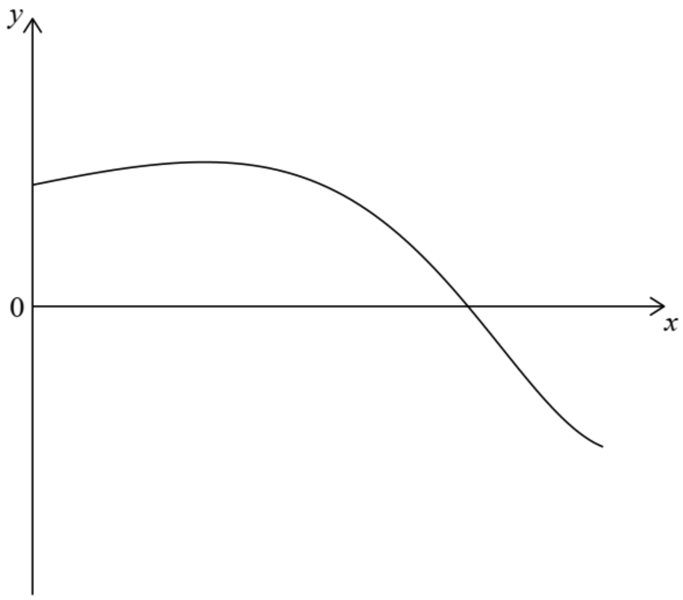
6. Let $f(x) = 4 - 2e^x$. The following diagram shows part of the graph of f.



(a) Find the x-intercept of the graph of f.

(b) Find the area of the region enclosed by the graph of f, the x-axis and the y-axis.

7. Let $f(x) = \sin(e^x)$ for $0 \leq x \leq 1.5$ The following diagram shows the graph of f.



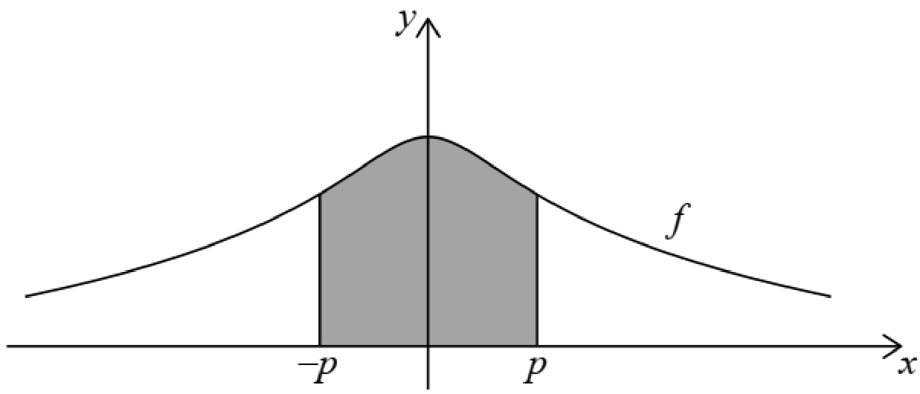
(a) Find the x-intercept of the graph of f.

(b) Find the area of the region enclosed by the graph of f, the x-axis and the y-axis.

8. Let $f(x) = 6 - \ln(x^2 + 2)$ for $x \in \mathbb{R}$. The graph of f passes through the point $(p, 4)$, where $p > 0$.

(a) Find the value of p.

(b) The following diagram shows part of the graph of f.



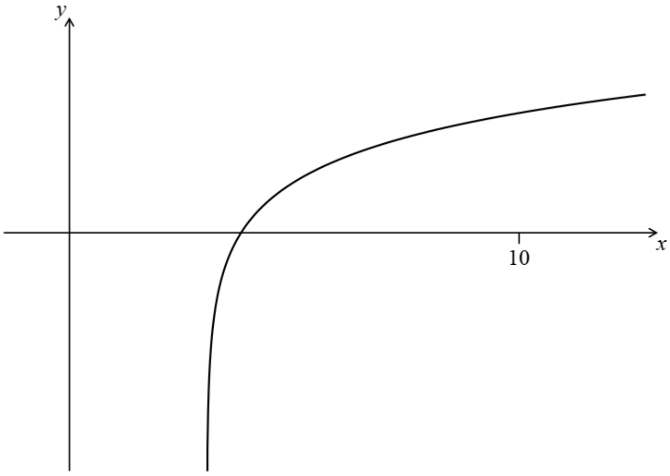
Find the area of the region enclosed by the graph of f, the x-axis and the lines $x = -p$ and $x = p$.

9. It is given that $\int_1^3 f(x)\,dx = 5$.

(a) Write down $\int_1^3 2f(x)\,dx$.

(b) Find the value of $\int_1^3 (3x^2 + f(x))\,dx$.

10. Let $f(x) = 2\ln(x - 3)$, for $x > 3$. The following diagram shows part of the graph of f.



(a) Find the equation of the vertical asymptote to the graph of f.

(b) Find the x-intercept of the graph of f.

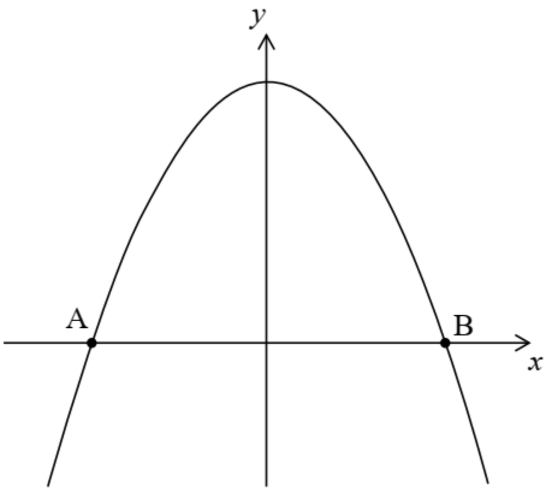
(c) Find the area of the region enclosed by the graph of f, the x-axis and the lines $x = 10$

11. Let $\int_1^5 3f(x)\,dx = 12$.

(a) Show that $\int_5^1 f(x)\,dx = -4$.

(b) Find the value of $\int_1^2 (x + f(x))\,dx + \int_2^5 (x + f(x))\,dx$.

12. Let $f(x) = 5 - x^2$. Part of the graph of f is shown in the following diagram.



The graph crosses the x-axis at the points A and B.

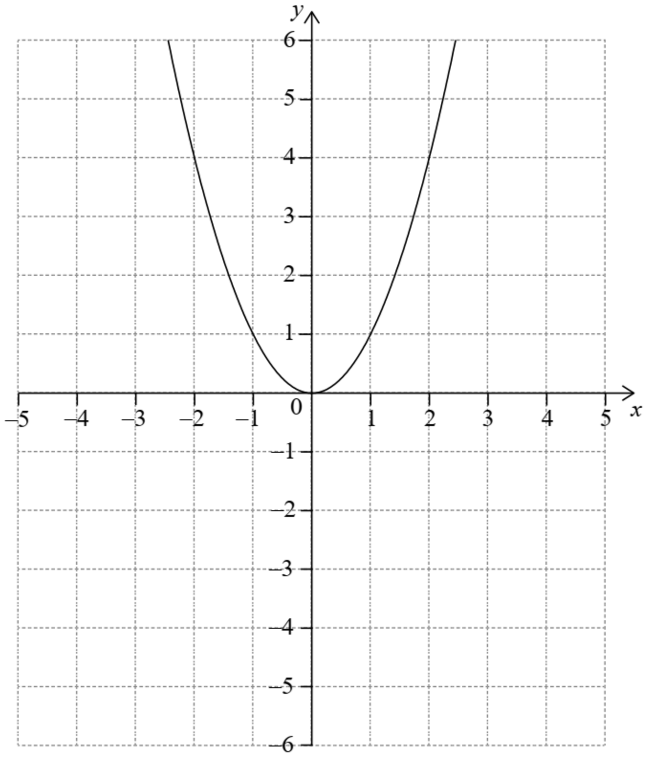
(a) Find the x-coordinate of A and of B

(b) Find the area of the region enclosed by the graph of f, and the x-axis

13. Let $g(x) = -(x-1)^2 + 5$.

(a) Write down the coordinates of the vertex of the graph of g.

Let $f(x) = x^2$. The following diagram shows part of the graph of f.



The graph of g intersects the graph of f at $x = -1$ and $x = 2$.

(b) On the grid above, sketch the graph of g for $-2 \le x \le 4$.

(c) Find the area of the region enclosed by the graphs of f and g.

14. Let $f(x) = xe^{-x}$ and $g(x) = -3f(x) + 1$.

The graphs of f and g intersect at $x = p$ and $x = q$, where $p < q$.

(a) Find the value of p and of q.

(b) Hence, find the area of the region enclosed by the graphs of f and g.

15. Let $f(x) = x^2$ and $g(x) = 3\ln(x+1)$, for $x > -1$.

(a) Solve $f(x) = g(x)$.

(b) Find the area of the region enclosed by the graphs of f and g.

16. Let $f(x) = \ln x$ and $g(x) = 3 + \ln\left(\frac{x}{2}\right)$, for $x > 0$.

The graph of g can be obtained from the graph of f by two transformations:

a horizontal stretch of scale factor q followed by

a translation of $\begin{pmatrix} h \\ k \end{pmatrix}$.

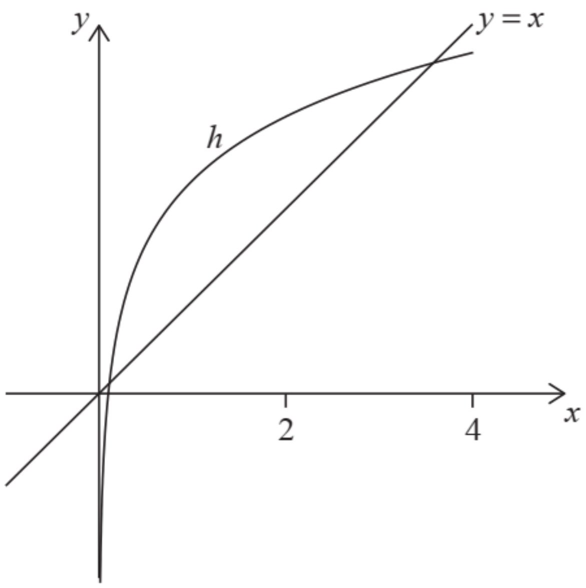
(a) Write down the value of

(i) q;

(ii) h;

(iii) k.

Let $h(x) = g(x) \times \cos(0.1x)$, for $0 < x < 4$. The following diagram shows the graph of h and the line $y = x$.



The graph of h intersects the graph of h^{-1} at two points. These points have x coordinates 0.111 and 3.31, correct to three significant figures.

(b) (i) Find $\int_{0.111}^{3.31} (h(x) - x)\,dx$.

(ii) Hence, find the area of the region enclosed by the graphs of h and h^{-1}.

(c) Let d be the vertical distance from a point on the graph of h to the line $y = x$. There is a point $\mathrm{P}(a, b)$ on the graph of h where d is a maximum.
Find the coordinates of P, where $0.111 < a < 3.31$.

17. Let $f(x) = xe^{-x}$ and $g(x) = -3f(x) + 1$.

The graphs of f and g intersect at $x = p$ and $x = q$, where $p < q$.

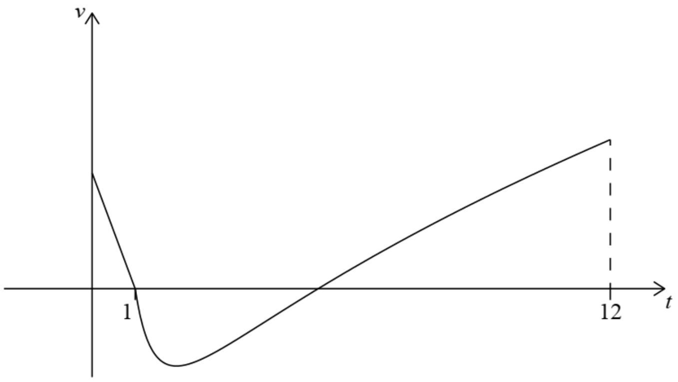
(a) Find the value of p and of q.

(b) Hence, find the area of the region enclosed by the graphs of f and g.

18. A particle P starts from a point A and moves along a horizontal straight line. Its velocity $v\ \mathrm{cm\,s^{-1}}$ after t seconds is given by

$$v(t) = \begin{cases} -2t + 2, \text{ for } 0 \le t \le 1 \\ 3\sqrt{t} + \dfrac{4}{t^2} - 7, \text{ for } 1 \le t \le 12 \end{cases}$$

The following diagram shows the graph of v.



(a) Find the initial velocity of P.

P is at rest when $t = 1$ and $t = p$.

(b) Find the value of p.

When $t = q$, the acceleration of P is zero.

(c) (i) Find the value of q.

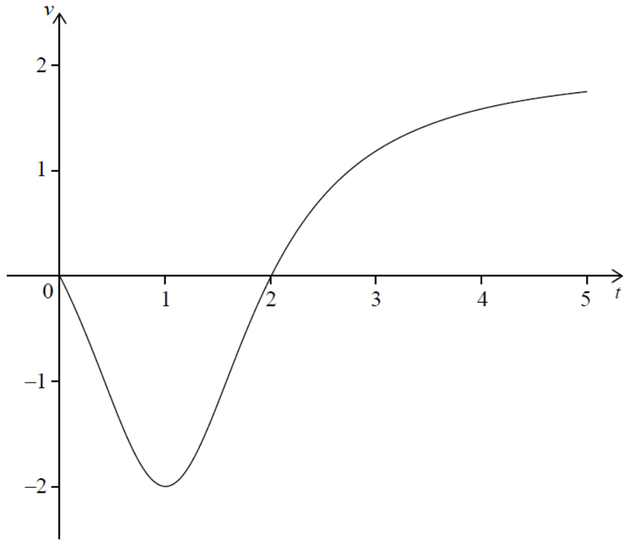
(ii) Hence, find the **speed** of P when $t = q$.

(d) (i) Find the total distance travelled by P between $t = 1$ and $t = p$.

(ii) Hence or otherwise, find the displacement of P from A when $t = p$.

19. Note: In this question, distance is in metres and time is in seconds.

A particle moves along a horizontal line starting at a fixed point A. The velocity v of the particle, at time t, is given by $v(t)=\dfrac{2t^2-4t}{t^2-2t+2}$, for $0 \le t \le 5$. The following diagram shows the graph of v



There are t-intercepts at $(0, 0)$ and $(2, 0)$.

Find the maximum distance of the particle from A during the time $0 \le t \le 5$ and justify your answer.

20. *In this question s represents displacement in metres and t represents time in seconds.*

The velocity v m s^{-1} of a moving body is given by $v = 40 - at$ where a is a non-zero constant.

(a) (i) If $s = 100$ when $t = 0$, find an expression for s in terms of a and t.

(ii) If $s = 0$ when $t = 0$, write down an expression for s in terms of a and t.

Trains approaching a station start to slow down when they pass a point P. As a train slows down, its velocity is given by $v = 40 - at$, where $t = 0$ at P. The station is 500 m from P.

(b) A train M slows down so that it comes to a stop at the station.

(i) Find the time it takes train M to come to a stop, giving your answer in terms of a.

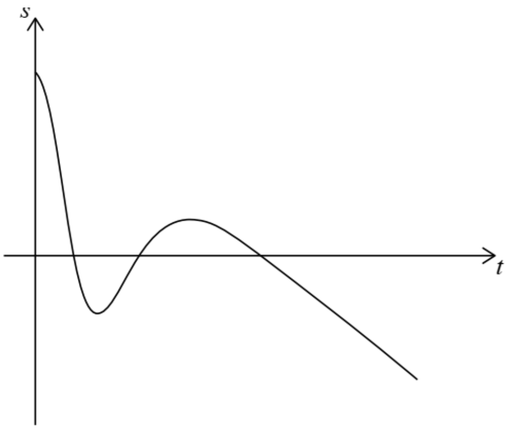
(ii) Hence show that $a = \dfrac{8}{5}$.

(c) For a different train N, the value of a is 4.
Show that this train will stop **before** it reaches the station.

21 The velocity v of a particle at time t is given by $v = e^{-2t} + 12t$. The displacement of the particle at time t is s. Given that $s = 2$ when $t = 0$, express s in terms of t.

22. **In this question distance is in centimetres and time is in seconds.**

Particle A is moving along a straight line such that its displacement from a point P, after t seconds, is given by $s_A = 15 - t - 6t^3 e^{-0.8t}$, $0 \le t \le 25$. This is shown in the following diagram.



(a) Find the initial displacement of particle A from point P.

(b) Find the value of t when particle A first reaches point P.

(c) Find the value of t when particle A first changes direction.

(d) Find the total distance travelled by particle A in the first 3 seconds.

Another particle, B, moves along the same line, starting at the same time as particle A. The velocity of particle B is given by $v_B = 8 - 2t$, $0 \le t \le 25$.

(e) (i) Given that particles A and B start at the same point, find the displacement function s_B for particle B.

(ii) Find the other value of t when particles A and B meet.

23. A particle moves along a straight line so that its velocity, $v \text{ m s}^{-1}$, after t seconds is given by $v(t) = 1.4^t - 2.7$, for $0 \le t \le 5$.

(a) Find when the particle is at rest.

(b) Find the acceleration of the particle when $t = 2$.

(c) Find the total distance travelled by the particle.

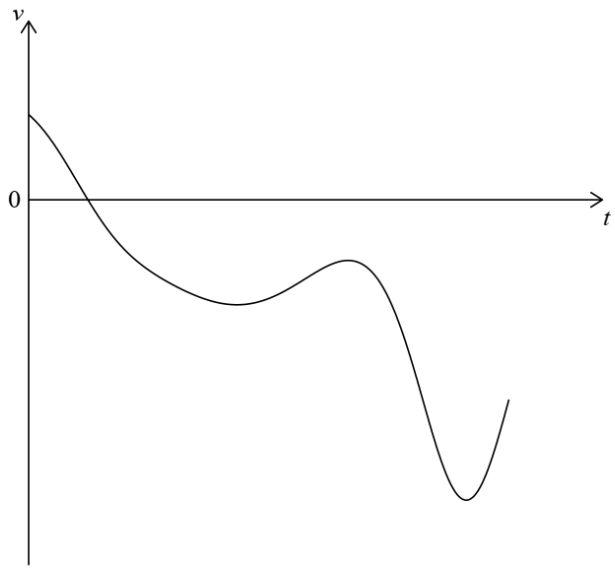
24. A particle moves in a straight line. Its velocity $v\,\mathrm{m\,s^{-1}}$ after t seconds is given by

$$v = 6t - 6, \text{ for } 0 \le t \le 2.$$

After p seconds, the particle is $2\,\mathrm{m}$ from its initial position. Find the possible values of p.

25. A particle P moves along a straight line. The velocity $v\,\mathrm{m\,s^{-1}}$ of P after t seconds is given by $v(t) = 7\cos t - 5t^{\cos t}$, for $0 \le t \le 7$.

The following diagram shows the graph of v.



(a) Find the initial velocity of P.

(b) Find the maximum speed of P.

(c) Write down the number of times that the acceleration of P is $0\,\mathrm{m\,s^{-2}}$.

(d) Find the acceleration of P when it changes direction.

(e) Find the total distance travelled by P.

26. **Note: In this question, distance is in metres and time is in seconds.**

A particle P moves in a straight line for five seconds. Its acceleration at time t is given by $a = 3t^2 - 14t + 8$, for $0 \le t \le 5$.

(a) Write down the values of t when $a = 0$.

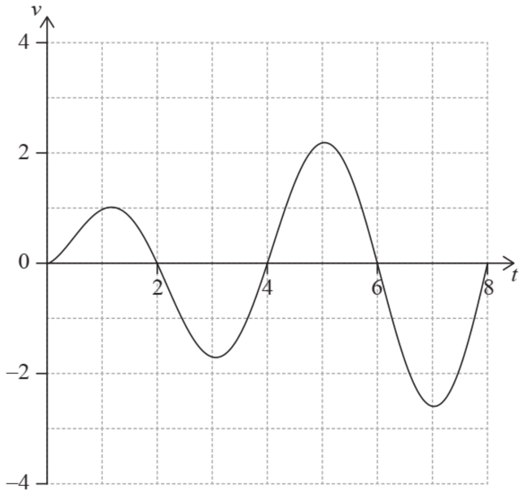
(b) Hence or otherwise, find all possible values of t for which the velocity of P is decreasing.

When $t = 0$, the velocity of P is $3\,\mathrm{m\,s^{-1}}$.

(c) Find an expression for the velocity of P at time t.

(d) Find the total distance travelled by P when its velocity is increasing.

27. A particle P moves along a straight line. Its velocity $v_P \text{ m s}^{-1}$ after t seconds is given by $v_P = \sqrt{t}\sin\left(\frac{\pi}{2}t\right)$, for $0 \le t \le 8$. The following diagram shows the graph of v_P.



(a) (i) Write down the first value of t at which P changes direction.

(ii) Find the **total** distance travelled by P, for $0 \le t \le 8$.

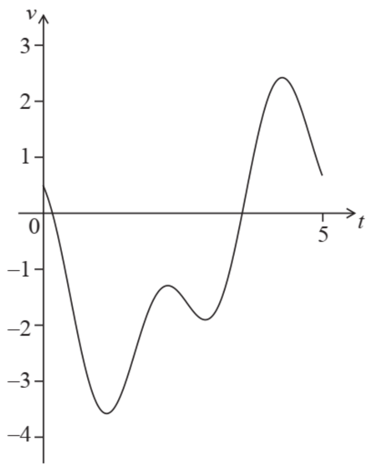
(b) A second particle Q also moves along a straight line. Its velocity, $v_Q \text{ m s}^{-1}$ after t seconds is given by $v_Q = \sqrt{t}$ for $0 \le t \le 8$. After k seconds Q has travelled the same total distance as P.

Find k.

28. A particle P moves along a straight line so that its velocity, $v \text{ ms}^{-1}$, after t seconds, is given by $v = \cos 3t - 2\sin t - 0.5$, for $0 \le t \le 5$. The initial displacement of P from a fixed point O is 4 metres.

(a) Find the displacement of P from O after 5 seconds.

The following sketch shows the graph of v.



(b) Find when P is first at rest.

(c) Write down the number of times P changes direction.

(d) Find the acceleration of P after 3 seconds.

(e) Find the maximum speed of P.

Integration

Student name: ANSWERS Score: ________

1. Let $f(x)=\sqrt[3]{x^4}-\frac{1}{2}$.

Find $\int f(x)\,\mathrm{d}x$. $= \frac{3}{7}\,x^{\frac{7}{3}} - \frac{x}{2} + c$

2. Let $f'(x)=12x^2-2$.

Given that $f(-1)=1$, find $f(x)$. $f(x) = 4x^3 - 2x + 3$

3. The curve $y=f(x)$ passes through the point (2, 6).

Given that $\frac{\mathrm{d}y}{\mathrm{d}x}=3x^2-5$, find y in terms of x. $y = 4x^3 - 5x + 4$

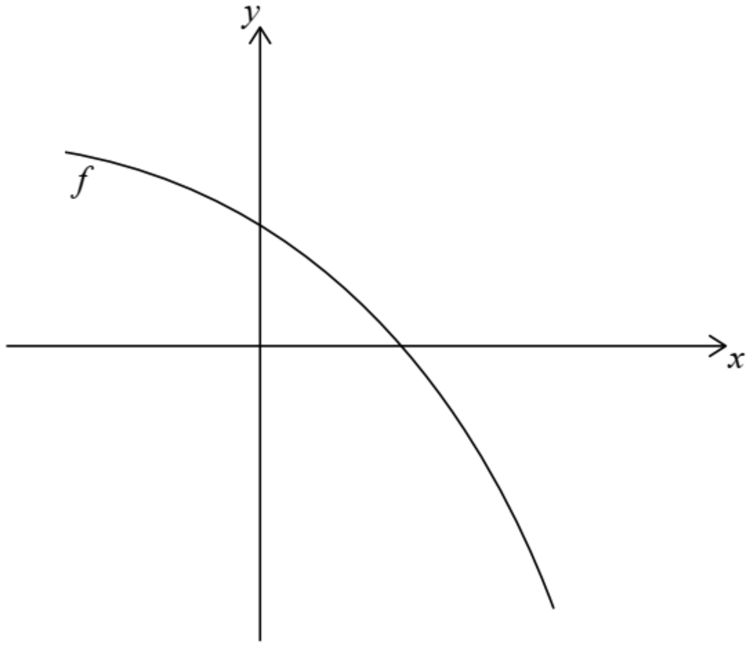
4. Let $f(x)=(3x+4)^5$. Find

$\int f(x)\,\mathrm{d}x$. $\frac{1}{18}(3x+4)^6 + C$

5. (a) Find $\int \frac{1}{2x+3}\,\mathrm{d}x$. $\frac{1}{2}\ln(2x+3) + C$

(b) Given that $\int_0^3 \frac{1}{2x+3}\,\mathrm{d}x = \ln\sqrt{P}$, find the value of P. $P = 3$

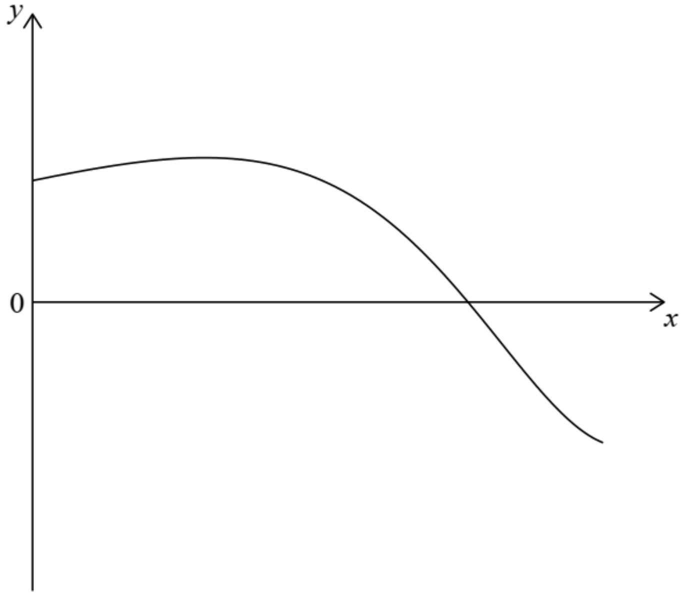
6. Let $f(x) = 4 - 2e^x$. The following diagram shows part of the graph of f.



(a) Find the x-intercept of the graph of f. ln 2 (exact); 0.693

(b) Find the area of the region enclosed by the graph of f, the x-axis and the y-axis.
4 ln 2 – 2; or 0.773

7. Let $f(x) = \sin(e^x)$ for $0 \leq x \leq 1.5$ The following diagram shows the graph of f.



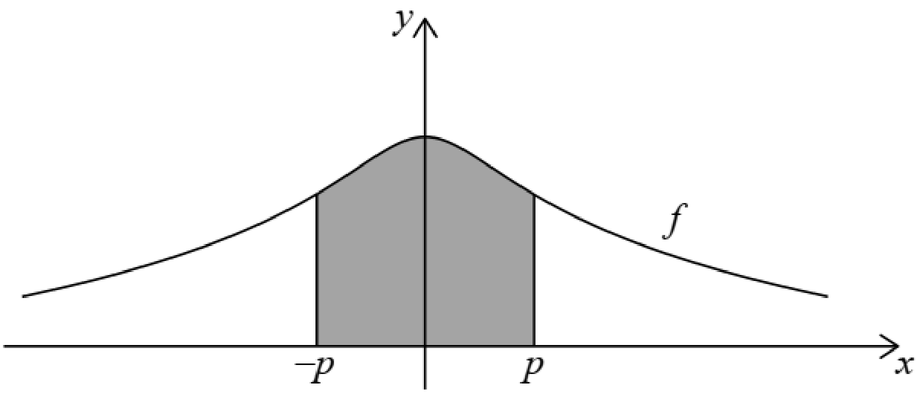
(a) Find the x-intercept of the graph of f. $\ln \pi$ (exact); 1.14

(b) Find the area of the region enclosed by the graph of f, the x-axis and the y-axis. 0.906

8. Let $f(x) = 6 - \ln(x^2 + 2)$ for $x \in \mathbb{R}$. The graph of f passes through the point $(p, 4)$, where $p > 0$.

(a) Find the value of p. $\sqrt{e^2 - 2}$

(b) The following diagram shows part of the graph of f.



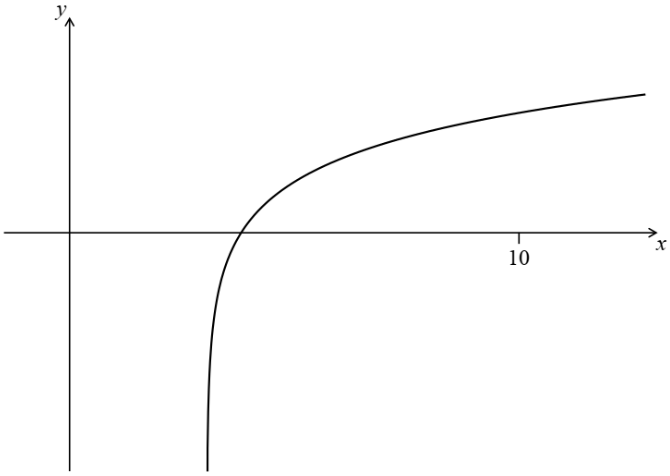
Find the area of the region enclosed by the graph of f, the x-axis and the lines $x = -p$ and $x = p$. 22.1

9. It is given that $\int_1^3 f(x)\,dx = 5$.

(a) Write down $\int_1^3 2f(x)\,dx$. 10

(b) Find the value of $\int_1^3 (3x^2 + f(x))\,dx$. 31

10. Let $f(x) = 2 \ln (x - 3)$, for $x > 3$. The following diagram shows part of the graph of f.



(a) Find the equation of the vertical asymptote to the graph of f. $x = 3$

(b) Find the x-intercept of the graph of f. $x = 4$

(c) Find the area of the region enclosed by the graph of f, the x-axis and the lines $x = 10$

15.2

11. Let $\int_1^5 3f(x)\,dx = 12$.

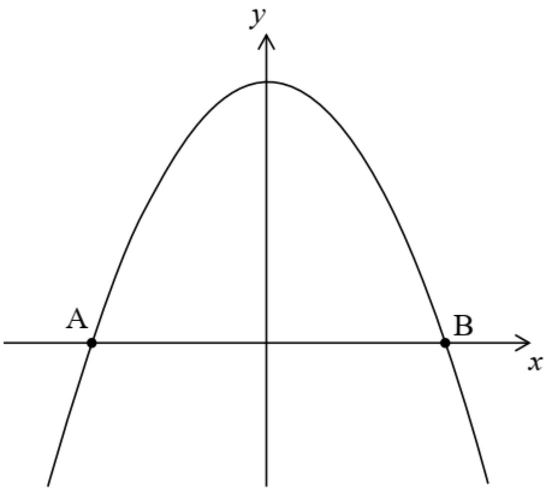
$$\int_1^5 f(x)dx = \int_1^5 \frac{3f(x)}{3}dx = \frac{12}{3} = 4$$

(a) Show that $\int_5^1 f(x)\,dx = -4$.

$$\int_5^1 f(x)dx = -\int_1^5 f(x)dx = -4$$

(b) Find the value of $\int_1^2 (x + f(x))\,dx + \int_2^5 (x + f(x))\,dx$. 16

12. Let $f(x) = 5 - x^2$. Part of the graph of f is shown in the following diagram.



The graph crosses the x-axis at the points A and B.

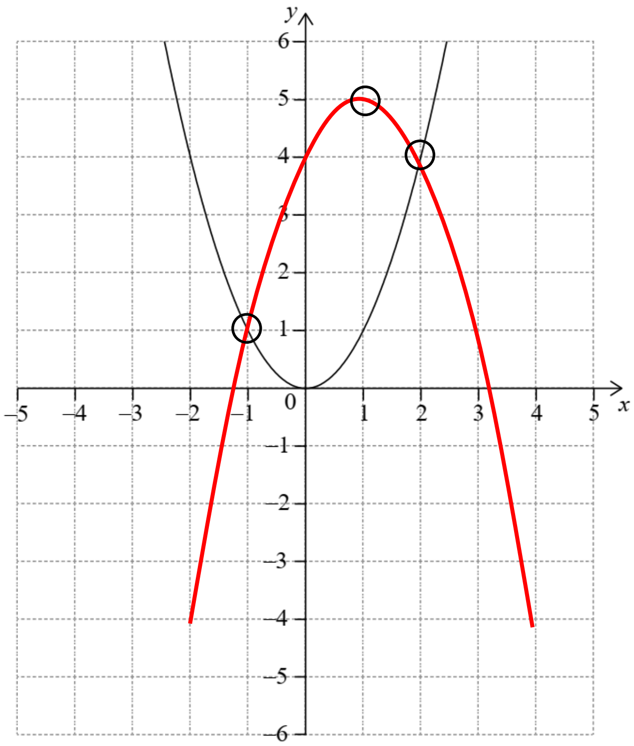
(a) Find the x-coordinate of A and of B $x = \pm\sqrt{5}$

(b) Find the area of the region enclosed by the graph of f, and the x-axis $\frac{20\sqrt{5}}{3}$; 14.9

13. Let $g(x) = -(x-1)^2 + 5$.

(a) Write down the coordinates of the vertex of the graph of g. (1, 5)

Let $f(x) = x^2$. The following diagram shows part of the graph of f.



The graph of g intersects the graph of f at $x = -1$ and $x = 2$.

(b) On the grid above, sketch the graph of g for $-2 \le x \le 4$.

(c) Find the area of the region enclosed by the graphs of f and g. area = 9

14. Let $f(x) = xe^{-x}$ and $g(x) = -3f(x) + 1$.

The graphs of f and g intersect at $x = p$ and $x = q$, where $p < q$.

(a) Find the value of p and of q. $p = 0.537$; $q = 2.15$

(b) Hence, find the area of the region enclosed by the graphs of f and g. area = 0.538

15. Let $f(x) = x^2$ and $g(x) = 3\ln(x+1)$, for $x > -1$.

(a) Solve $f(x) = g(x)$. $x = 0$; $x = 1.74$

(b) Find the area of the region enclosed by the graphs of f and g. area = 1.31

16. Let $f(x) = \ln x$ and $g(x) = 3 + \ln\left(\frac{x}{2}\right)$, for $x > 0$.

The graph of g can be obtained from the graph of f by two transformations:

a horizontal stretch of scale factor q followed by

a translation of $\begin{pmatrix} h \\ k \end{pmatrix}$.

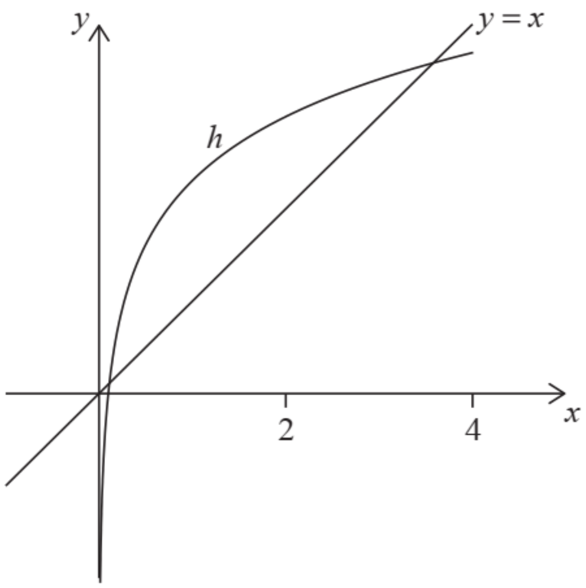
(a) Write down the value of

(i) q; $q = 2$

(ii) h; $h = 0$

(iii) k. $k = 3$

Let $h(x) = g(x) \times \cos(0.1x)$, for $0 < x < 4$. The following diagram shows the graph of h and the line $y = x$.



The graph of h intersects the graph of h^{-1} at two points. These points have x coordinates 0.111 and 3.31, correct to three significant figures.

(b) (i) Find $\int_{0.111}^{3.31} (h(x) - x)\,\mathrm{d}x$. 2.72

(ii) Hence, find the area of the region enclosed by the graphs of h and h^{-1}.

5.45

(c) Let d be the vertical distance from a point on the graph of h to the line $y = x$. There is a point $\mathrm{P}(a, b)$ on the graph of h where d is a maximum.
Find the coordinates of P, where $0.111 < a < 3.31$. $a = 0.974$; $b = 2.27$

17. Let $f(x) = xe^{-x}$ and $g(x) = -3f(x) + 1$.

The graphs of f and g intersect at $x = p$ and $x = q$, where $p < q$.

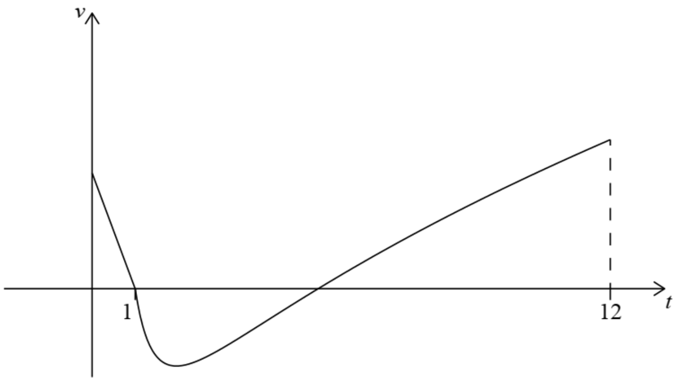
(a) Find the value of p and of q. $p = 0.357$; $q = 2.15$

(b) Hence, find the area of the region enclosed by the graphs of f and g. 0.538

18. A particle P starts from a point A and moves along a horizontal straight line. Its velocity $v \text{ cm s}^{-1}$ after t seconds is given by

$$v(t) = \begin{cases} -2t + 2, \text{ for } 0 \le t \le 1 \\ 3\sqrt{t} + \dfrac{4}{t^2} - 7, \text{ for } 1 \le t \le 12 \end{cases}$$

The following diagram shows the graph of v.



(a) Find the initial velocity of P. 2

P is at rest when $t = 1$ and $t = p$.

(b) Find the value of p. $p = 5.22$

When $t = q$, the acceleration of P is zero.

(c) (i) Find the value of q. $q = 1.95$

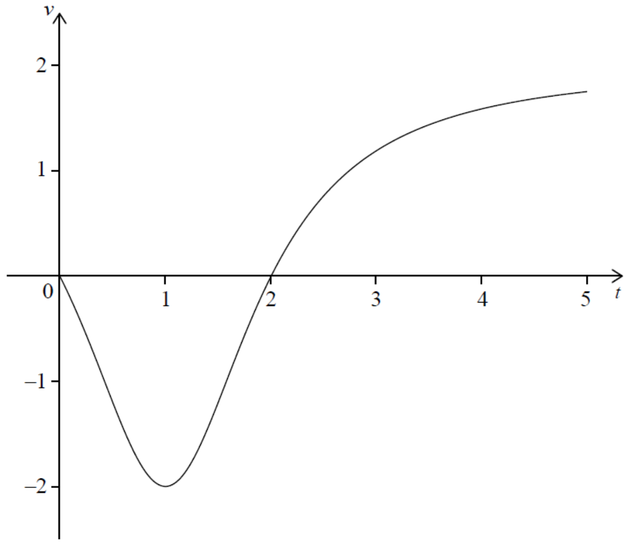
(ii) Hence, find the **speed** of P when $t = q$. 1.76

(d) (i) Find the total distance travelled by P between $t = 1$ and $t = p$. 4.45 cm

(ii) Hence or otherwise, find the displacement of P from A when $t = p$. -3.45 cm

19. Note: In this question, distance is in metres and time is in seconds.

A particle moves along a horizontal line starting at a fixed point A. The velocity v of the particle, at time t, is given by $v(t) = \dfrac{2t^2 - 4t}{t^2 - 2t + 2}$, for $0 \le t \le 5$. The following diagram shows the graph of v



There are t-intercepts at $(0, 0)$ and $(2, 0)$.

Find the maximum distance of the particle from A during the time $0 \le t \le 5$ and justify your answer. 2.28m

20. *In this question s represents displacement in metres and t represents time in seconds.*

The velocity v m s^{-1} of a moving body is given by $v = 40 - at$ where a is a non-zero constant.

(a) (i) If $s = 100$ when $t = 0$, find an expression for s in terms of a and t. $s = 40t - \frac{1}{2}at^2 + 100$

(ii) If $s = 0$ when $t = 0$, write down an expression for s in terms of a and t. $s = 40t - \frac{1}{2}at^2$

Trains approaching a station start to slow down when they pass a point P. As a train slows down, its velocity is given by $v = 40 - at$, where $t = 0$ at P. The station is 500 m from P.

(b) A train M slows down so that it comes to a stop at the station.

(i) Find the time it takes train M to come to a stop, giving your answer in terms of a. $t = \frac{40}{a}$

(ii) Hence show that $a = \dfrac{8}{5}$.

$500 = 40\left(\frac{40}{a}\right) - \frac{1}{2}a\left(\frac{40}{a}\right)^2$

$500 = \frac{800}{a}$

Stops when $v = 0$

$v = 40 - at$

$0 = 40 - 4t$

$t = 10$

(c) For a different train N, the value of a is 4.
Show that this train will stop **before** it reaches the station.

$s = 40(10) - \frac{1}{2}(4)(10)^2$

$s = 200$

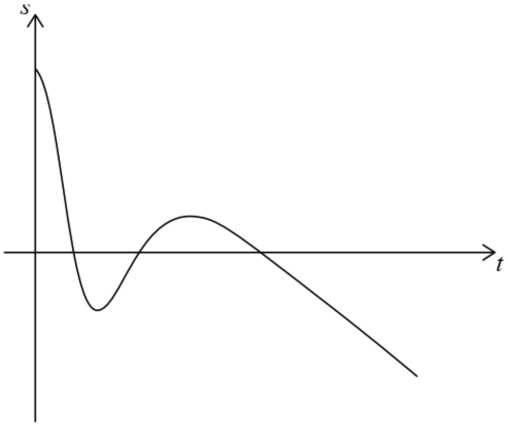
200 < 500 So, It will stop before it reaches the station

21. The velocity v of a particle at time t is given by $v = e^{-2t} + 12t$. The displacement of the particle at time t is s. Given that $s = 2$ when $t = 0$, express s in terms of t.

$s = -0.5e^{-2t} + 6t^2 + 2.5$ $\quad$ $s = -0.5e^{-2t} + 6t^2 + 2.5$

22. **In this question distance is in centimetres and time is in seconds.**

Particle A is moving along a straight line such that its displacement from a point P, after t seconds, is given by $s_A = 15 - t - 6t^3e^{-0.8t}$, $0 \le t \le 25$. This is shown in the following diagram.



(a) Find the initial displacement of particle A from point P. 15 cm

(b) Find the value of t when particle A first reaches point P. 2.47 s

(c) Find the value of t when particle A first changes direction. 4.08 s

(d) Find the total distance travelled by particle A in the first 3 seconds. 17.7 cm

Another particle, B, moves along the same line, starting at the same time as particle A. The velocity of particle B is given by $v_B = 8 - 2t$, $0 \le t \le 25$.

(e) (i) Given that particles A and B start at the same point, find the displacement function s_B for particle B. $S_B(t) = 8t - 6t^2 + 15$

(ii) Find the other value of t when particles A and B meet. 9.30 s

23. A particle moves along a straight line so that its velocity, v m s^{-1}, after t seconds is given by $v(t) = 1.4^t - 2.7$, for $0 \le t \le 5$.

(a) Find when the particle is at rest. $t = \log_{1.4} 2.7$ (s) exact or $t \approx 2.95$ (s)

(b) Find the acceleration of the particle when $t = 2$. $a(2) = 1.96\ln 1.4$ (exact), $a(2) = 0.659$

(c) Find the total distance travelled by the particle. 5.35 m

24. A particle moves in a straight line. Its velocity $v\,\mathrm{m\,s^{-1}}$ after t seconds is given by

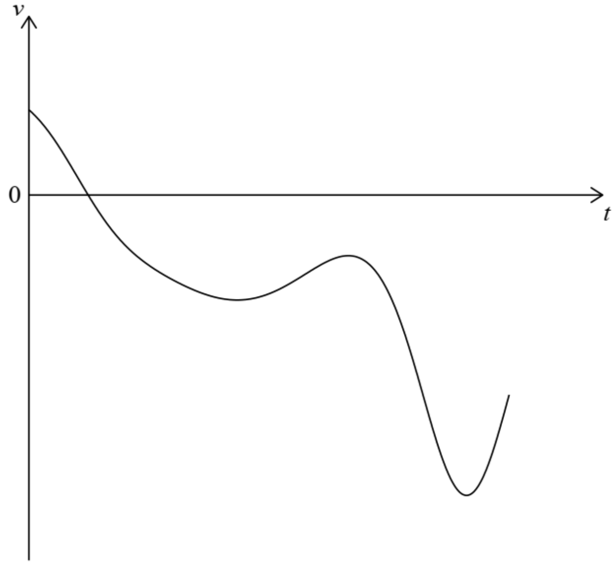
$$v = 6t - 6, \text{ for } 0 \le t \le 2.$$

After p seconds, the particle is $2\,\mathrm{m}$ from its initial position. Find the possible values of p.

$p = 0.423$ or $p = 1.58$

25. A particle P moves along a straight line. The velocity $v\,\mathrm{m\,s^{-1}}$ of P after t seconds is given by $v(t) = 7\cos t - 5t^{\cos t}$, for $0 \le t \le 7$.

The following diagram shows the graph of v.



(a) Find the initial velocity of P. $v = 7$

(b) Find the maximum speed of P. 24.7

(c) Write down the number of times that the acceleration of P is $0\,\mathrm{m\,s^{-2}}$. 3

(d) Find the acceleration of P when it changes direction. $a = -9.25$

(e) Find the total distance travelled by P. 63.9

26 **Note: In this question, distance is in metres and time is in seconds.**

A particle P moves in a straight line for five seconds. Its acceleration at time t is given by $a = 3t^2 - 14t + 8$, for $0 \le t \le 5$.

(a) Write down the values of t when $a = 0$. $t = \frac{2}{3}$; $t = 4$

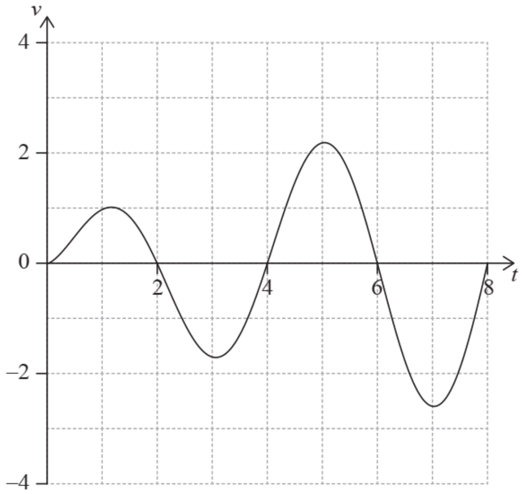
(b) Hence or otherwise, find all possible values of t for which the velocity of P is decreasing. 63.9

When $t = 0$, the velocity of P is $3\,\mathrm{m\,s^{-1}}$.

(c) Find an expression for the velocity of P at time t. $v = t^3 - 7t^2 + 8t + 3$

(d) Find the total distance travelled by P when its velocity is increasing. 14.2

27. A particle P moves along a straight line. Its velocity $v_P \text{ m s}^{-1}$ after t seconds is given by $v_P = \sqrt{t}\sin\left(\frac{\pi}{2}t\right)$, for $0 \le t \le 8$. The following diagram shows the graph of v_P.



(a) (i) Write down the first value of t at which P changes direction. $t = 2$

(ii) Find the **total** distance travelled by P, for $0 \le t \le 8$. 9.65 m

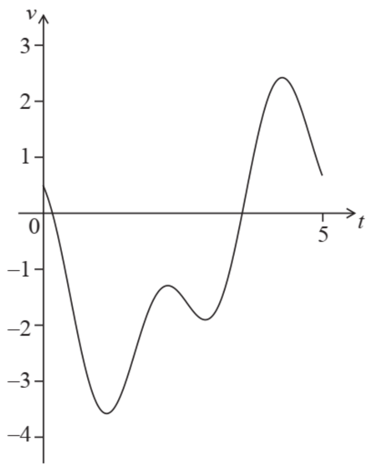
(b) A second particle Q also moves along a straight line. Its velocity, $v_Q \text{ m s}^{-1}$ after t seconds is given by $v_Q = \sqrt{t}$ for $0 \le t \le 8$. After k seconds Q has travelled the same total distance as P.

Find k. 5.94 seconds

28. A particle P moves along a straight line so that its velocity, $v \text{ ms}^{-1}$, after t seconds, is given by $v = \cos 3t - 2\sin t - 0.5$, for $0 \le t \le 5$. The initial displacement of P from a fixed point O is 4 metres.

(a) Find the displacement of P from O after 5 seconds. 0.284 m

The following sketch shows the graph of v.



(b) Find when P is first at rest. 0.180 s

(c) Write down the number of times P changes direction. 2 times

(d) Find the acceleration of P after 3 seconds. 0.744

(e) Find the maximum speed of P. 3.28